THE APPLICATION OF HOLOGRAPHIC INTERFEROMETRY TO THE DETERMINATION OF ASYMMETRIC THREE-DIMENSIONAL DENSITY FIELDS IN FREE JET FLOW

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THESIS

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Submitted in partial fulfillment of the
requirements of the degree of

DOCTOR OF PHILOSOPHY

from the

NAVAL POSTGRADUATE SCHOOL

June 1970

ABSTRACT

The successful application of holographic interferometry, and an associated mathematical reduction process, to the determination of an asymmetric three-dimensional density field of an aerodynamic phenomenon is reported.

An integral inversion method from the field of plasma physics has been computerized, extensively evaluated and applied to the determination of functions, both axisymmetric and asymmetric, which simulate aerodynamic density fields.

The application of holographic interferometry has been extended to provide multiple holograms about a test region, with sufficient coverage to provide interferometric data for the successful solution of the density field.

The analytical and experimental methods developed were applied to an experimental axisymmetric test field, the supersonic flow from a free jet, and shown to be comparable to a previous solution obtained by the Abel inversion method. Further, the free jet was tilted to provide a test field which was asymmetric in the plane of solution. Comparison of the resulting asymmetric solution was shown to be self-consistent with the previously obtained axisymmetric solution.

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I. INTRODUCTION

Holography has enabled the aerodynamicist to "freeze" the interferometric view of a transient phenomenon and subsequently to view the field in three dimensions, as through a "window." This new capability has indicated the possibility of quantitatively determining the density in such a field, with no symmetric restrictions on its form.

To make quantitative determinations of asymmetric density fields, it was necessary to:

1) Invert the fringe number functions that describe the interferometric data,

$$g(y', \xi) = \int_{x_1'}^{x_2'} f(x, y)\, dx'$$

to obtain the asymmetric density function f from within the integral. In the search for such an inversion process the possibility of using an asymmetric line integral inversion scheme which had been developed in the field of plasma emissivity [Maldonado, 1966], became evident. The method was evaluated by extensive computerized testing on density functions found in aerodynamics, including shock waves. The inversion method was found to be quite accurate, giving results which were generally within one percent of the test function.

2) Obtain holographic interferograms of sufficient angular coverage about a test region to provide the interferometric data required by the inversion process. To obtain the necessary experimental data, a holographic

work table was constructed and used to design the necessary optical arrangement. The capability of achieving multiple Q-switched holographic interferograms about a test region, with no opaque objects present, was demonstrated.

3) Apply real holographic interferometric data to an inversion process in order to demonstrate the practicality of the technique. A stepwise evaluation of the system included an axisymmetric evaluation of a free jet for comparison with the previously available solution of Winckler [1948]. Having attained a solution for the three-dimensional axisymmetric field, the asymmetric capability of the technique was tested by tilting the free jet, which destroyed the axisymmetry in the plane of inversion. The resulting data were inverted asymmetrically and compared with the axisymmetrically measured density. The result is shown to be a self-consistent comparison of the asymmetric inversion.

II. THEORETRICAL ANALYSIS

A. THE INTERFEROMETRY OF AERODYNAMIC FIELDS

Holography has opened a new branch of interferometry for aerodynamic research. Ordinary Mach-Zehnder type interferograms have been restricted to the analysis of two-dimensional or axisymmetric flow fields. Such interferograms are able to examine only the light beam which has passed through the test region from a single direction. Holograms are able to reproduce a light wave in both amplitude and phase, and further to

simultaneously reproduce two such light waves which have occurred at different times. The resulting comparison of two reproduced light waves yields an interferogram which allows the analysis of the light which has passed through a test region from many different directions.

By interfering two coherent light waves with each other, interferometry compares their phases, and thus provides a measured comparison of their integrated index of refraction over their respective distances of travel from a common source to the film. This relative phase, expressed in multiples of wavelengths and commonly called fringe number, is described by the equation:

$$g = \frac{1}{\lambda} \int_{L_1}^{L_2} \left[n(x,y,z) - n_o(x,y,z) \right] dl \qquad (1)$$

where: λ = the wavelength of the light at the film,

n = the index of refraction seen by the traveling light wave,

n_o = the index of refraction seen by the wave of the comparison scene,

L_2-L_1= the total distance of travel for each of the waves.

The index of refraction of a gas is well represented as a function of the gas density [Liepmann and Roshko, 1957] by the first two terms of a Taylor expansion:

$$n = 1 + \beta \frac{\rho}{\rho_s} + \ldots \qquad (2)$$

where: ρ_s = the gas density at which β is measured,

β = the measured coefficient.

For air at 0°C. and 760 mm. Hg., $\rho_s = .0012929$ gm/cc., and for deep red light, $\beta = .000291$. Thus one can rewrite equation (1):

$$g = \frac{\beta}{\rho_s \lambda} \int_{L_1}^{L_2} \left[\rho (x,y,z) - \rho_o (x,y,z) \right] dl \qquad (3)$$

Equation (3) is a line integral equation for the unknown density. With sufficient evaluation of the function g (a function of three spatial variables which one obtains from the interferograms), one can invert the equation to obtain the density function.

1. The Mach-Zehnder Interferometer

Figure (1) is the schematic arrangement of a typical Mach-Zehnder interferometer [Ladenberg, ed. 1954]. The two light waves which are compared travel paths A and B respectively. Alignment of the instrument is critical to within the order of a wavelength of light, and the mirrors and lenses must be distortion free to one-tenth of that value. The chief practical disadvantage, however, is that one can evaluate the light beam from only one direction at a time through the test section.

By properly aligning the components with a uniform density field in the test section, one can exactly recombine the two light beams A and B at the film plane of the camera. Then with a flow in the test section, any phase differences recorded are those due to variations in the test section density. Such an interferogram is called "infinite fringe."

If one purposely misaligns one beam with the other (by rotating a mirror slightly) a deliberate fringe pattern is recorded. Variation in test section density is then recorded as a variation in the regularity of the

fringe pattern. This second variety of interferogram is called "finite fringe."

2. Holography

First reported by Denis Gabor [1948, 1949, 1951], holography is itself an interferometric process. By combining two coherent beams directly on a photographic film (Figures 2, 2a), one records a microscopically small interference pattern. Positions on the film where the two waves are in phase will record darkly. Positions where they are out of phase will remain light. When the developed photographic plate is reilluminated by the reference beam (Figures 3, 3a), the interference pattern acts as a very complex diffraction grating and diffracts three beams; these are the zero order, or transmitted illumination beam, and the positive and negative first orders, which are re-creations of the original object beam.

Mathematically, one can represent the phenomenon as the combination of any two general waves which obey the scalar wave equation [Brandt, 1968]. Let a wave be described as follows:

$\tilde{E}(\bar{r},t) = \tilde{A}(\bar{r})\, e^{i\omega t}$ is the complex wave, where (4)

$\tilde{A}(\bar{r}) = A(\bar{r})\, e^{-i\Psi(\bar{r})}$ is the spatial portion, (5)

$A(\bar{r}) = a\, f(\bar{r})$ is the spatial amplitude, (6)

$\Psi(\bar{r})$ is the spatial phase function,

a is the reference amplitude,

$f(\bar{r})$ is the spatial variation of amplitude, and

$\bar{r}$ is a position vector.

Since photographic film measures only the time averaged intensity over relatively many periods of the wave, one need not be concerned with the temporal portion $e^{i\omega t}$ of the wave. Denoting the object and reference light beams by the subscripts o and r, one can represent the combination of both beams at the photographic plate as:

$$\tilde{A} = \tilde{A}_r + \tilde{A}_o = A_r e^{-i\Psi_r} + A_o e^{-i\Psi_o} \tag{7}$$

The intensity recorded will be:

$$I = \tilde{A}\tilde{A}^* = (\tilde{A}_r + \tilde{A}_o)(\tilde{A}_r + \tilde{A}_o)^* = |A_r|^2 + |A_o|^2 + \tilde{A}_r^*\tilde{A}_o + \tilde{A}_r\tilde{A}_o^* \tag{8}$$

where * denotes complex conjugate. Assuming a linear ratio of intensity to transmittance of the developed hologram, the transmittance is:

$$T = 1 - T_1 I = 1 - T_1(|A_r|^2 + |A_o|^2) - T_1(\tilde{A}_r^*\tilde{A}_o) - T_1(\tilde{A}_r\tilde{A}_o^*) \tag{9}$$

where T_1 is the transmittance corresponding to unit intensity. Upon reillumination by the reference beam, the transmitted wave is:

$$\tilde{A}_r T = \left[1 - T_1(|A_r|^2 + |A_o|^2)\right]\tilde{A}_r - \left[T_1|A_r|^2\right]\tilde{A}_o - (T\tilde{A}_r\tilde{A}_r)\tilde{A}_o^* \tag{10}$$

where the quantities in square brackets represent attenuation. The first term represents the transmission of the reilluminating reference beam, the second term represents the production of an attenuated object beam, and the third term represents a modified reconstruction of the complex conjugate of the object beam (Figure 3). The modification of the conjugate object wave depends upon the nature and direction of the reference beam, but for simple reference waveforms there is usually only a magnification

and angular displacement to the opposite side of the reference wave. The negative sign on the reconstructed wave amplitude is immaterial, since a negative amplitude on a continuous wave simply represents a 180° phase change.

If the reillumination beam is different from the original reference beam, the reconstructed diffraction beams are distorted accordingly. For the most part, this distortion is due to different source location or different wavelength of light, which create magnification and angular displacement of the object waves.

For laboratory use, the reconstructed holographic image can be considered an almost exact duplication of the original.

3. Holographic Interferometry

The application of holography to interferometry [Heflinger, et. al., 1966] derives from the fact that the holographic film records the diffraction pattern almost linearly. When double exposed, the hologram records two diffraction patterns superimposed upon one another. Therefore, when a double exposed hologram is reilluminated, each recorded diffraction pattern will diffract its own first order beams. An observer will see both recorded scenes simultaneously. One may simply replace A_o by $A_{o1} + A_{o2}$ in equations (8-10). To produce a holographic interferogram of a test section as in Figure 2, one exposes the hologram for one-half the exposure time with no flow in the test section (the uniform field provides the

comparison beam); then completes the exposure with the subject flow field present. The two reconstructed waves will interfere with each other in much the same manner as do the two waves of the Mach-Zehnder in the infinite fringe configuration.

If, instead of making a second exposure of the hologram, one replaces it exactly in position and reilluminates both the hologram and the object, one can again compare the two waves, except that in this case one of the object scenes is real and the other is a reconstructed virtual image. Such real-time holographic interferometry is called the "live" fringe technique.

The chief advantage of either method is that, except for differences in the test section, both the test and the comparison beams have traveled through exactly the same optical regions. Optical components are then automatically matched. In practice rather crude optical components can be used with excellent results.

By a slight rotation of the hologram or of one of the mirrors between exposures, one can achieve the same finite fringe presentation as with the Mach-Zehnder.

a. Dark Field Interferograms

Dark field interferograms are produced when the test section does not contain any light scattering properties. The observer looking at the reconstructed image is unable to focus on anything other than the

source; objects in the test section, as well as regions of interference, can only be observed as shadows. Such interferograms correspond to those taken by ordinary methods with only one direction of light beam passage through the test field, corresponding to only one light source.

b. Light Field Interferograms

True three-dimensional interferograms are obtained when a diffuser plate is placed between the source and the test section. The object beam then becomes diffused through the test section and appears to the observer as a continuous background of source points against which the test section, and its interference pattern, become a silouette. Because each point of the diffuse plate acts as an individual source for any line of sight passing through it, equation (3) may be evaluated for any line which passes through both the diffuser plate and the hologram. A continuous evaluation of the function g can thus be provided as a function of position and angle. This function g, the fringe pattern, changes as the observer changes viewing aspect, but generally the fringe patterns cannot be localized by their parallax.

By arranging several holographic plates about a test section, as in Figure 4, one can obtain the fringe number function for a rather continuous segment of angular variation about the z axis of the field. The resulting array of integral values can be applied to the inversion of equation (3) to provide a solution for the density field in the test section.

B. THE INTEGRAL INVERSION

1. The Basic Equation to Invert

Equation (3), rewritten for a plane of constant z is:

$$g(y', \S, z_c) = Q \int_{x_1'}^{x_2'} f(x, y, z_c)\, dx' \tag{11}$$

where:

$$f(x, y, z_c) = \frac{\rho(x, y, z_c)}{\rho_\infty} - 1 \tag{12}$$

and:

$$Q = \frac{\rho_\infty \beta}{\rho_s \lambda} \tag{13}$$

x' and y' are measured in a coordinate system which is rotated by an angle § about the z axis (Figure 5).

2. Maldonado's Inversion

The integral inversion method utilized in this investigation was first reported by C. D. Maldonado et. al. in 1965 [1966; Olsen, 1968]. It was used for obtaining plasma emissivity within a particular region from the measured values of emission intensity measured from outside the region. The form of the equation resulting from such emissivity studies is identical to that of equation (11). The procedure involves the representation of the function f of equation (11) in a complete set of orthogonal functions, with the expansion coefficients evaluated by use of the orthogonality condition.

An earlier method involving series expansion of the function f had been reported in 1962 by S. I. Herlitz. The method applied to cases of asymmetric function of finite domain. Maldonado's method follows similar logic, but it is applicable to asymmetric functions of infinite domain.

The function f is assumed to be squared integrable over the infinite plane so that it may be expanded in a complete set of orthogonal functions. The selection of a suitable set of orthogonal functions is discussed in Appendix B.

a. The Analytical Procedure

The unknown function may be expanded in a special set of functions $U^{\pm m}_{m+2k}$ which are orthogonal with respect to the weighting function $e^{-(\alpha^2x^2+\alpha^2y^2)}$:

$$f(x,y)=\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\epsilon_m\left[C^{m}_{m+2k}(\alpha)U^{m}_{m+2k}(\alpha x,\alpha y)+C^{-m}_{m+2k}(\alpha)U^{-m}_{m+2k}(\alpha x,\alpha y)\right]\cdot e^{-(\alpha^2x^2+\alpha^2y^2)} \tag{14}$$

where $\epsilon_m=\frac{1}{2}$ for $m=0$, $\epsilon_m=1$ for $m=1,2,3\ldots$ and $C^{\pm m}_{m+2k}$ are the unknown complex coefficients of expansion. α is an arbitrary scale factor which may be considered the reciprocal of a non-dimensionalizing coefficient.

The functions $U^{\pm m}_{m+2k}$ are defined:

$$U^{\pm m}_{m+2k}(\alpha x,\alpha y)=(-1)^k\alpha\left[\frac{k!\,(\alpha^2x^2+\alpha^2y^2)^m}{\pi\,(m+k)!}\right]^{\frac{1}{2}}e^{\pm im\phi}\,L^{m}_{k}(\alpha^2x^2+\alpha^2y^2) \tag{15}$$

where: $\phi=\tan^{-1}\left(\frac{y}{x}\right)-\frac{\pi}{2}$, and L^{m}_{k} is the associated Laguerre polynomial:

$$L^{m}_{k}(\alpha^2x^2+\alpha^2y^2)=\sum_{s=0}^{k}\left[\frac{(m+k)!}{(k-s)!\,(m+s)!\,s!}\right]\cdot\left[(-1)\,(\alpha^2x^2+\alpha^2y^2)\right]^{s} \tag{16}$$

The function $U^{\pm m}_{m+2k}$ has a gauss transform:

$$I^{\pm m}_{m+2k}(\alpha y',\xi)=\int_{-\infty}^{\infty}U^{\pm m}_{m+2k}(\alpha x,\alpha y)\,e^{-\alpha^2x'^2}\,dx'=\frac{e^{\pm im\xi}\,H_{m+2k}\,\alpha y'}{\left[k!\,(m+k)!\right]^{1/2}\cdot 2^{m+2k}} \tag{16a}$$

where $H_{m+2k}(\alpha y')$ are Hermite polynomials of order m+2k. The particular advantage of the set of functions $U^{\pm m}_{m+2k}$ is that they are "invariant in form" to a rotation of coordinate system [Maldonado, 1965]. That is, they remain an orthogonal set under a rotation of the coordinate system. Observe that in the equation for the polynomial (15), the angle ϕ occurs only in the complex exponential term.

In terms of the expanded function f, and utilizing the transform relation of equation (16), equation (11) becomes:

$$g(y',\xi) = \sum_{m=0}^{\infty}\sum_{k=0}^{\infty} \epsilon_m \left[k!\,(m+k)!\,2^{2m+2k}\right]^{-\frac{1}{2}} \left[C^{m}_{m+2k}(\alpha)\,e^{im\xi} + C^{-m}_{m+2k}(\alpha)\,e^{-im\xi}\right] H_{m+2k}(\alpha)\,e^{-\alpha^2 y'^2} \tag{17}$$

Equation (17) is subject to the following orthogonality condition:

$$\int_{-\pi}^{\pi} e^{\pm im\xi}\, e^{\mp in\xi}\, d\xi \int_{-\infty}^{\infty} H_{m+2k}(\alpha y')\, H_{n+2\ell}(\alpha y')\, e^{-\alpha^2 y'^2}\, dy' = \frac{2\pi^{\frac{3}{2}}}{\alpha}\left[(m+2k)!\,(n+2\ell)!\,2^{m+2k}\,2^{n+2\ell}\,\delta_{mn}\,\delta_{(m+2k)(n+2\ell)}\right] \tag{18}$$

Where δ is the kroneker delta function. The solution of equation (18) applied to equation (17) leads to the expansion coefficients:

$$C^{\pm m}_{m+2k}\,\alpha = \frac{\alpha}{2\pi^{3/2}}\left[\frac{k!\,(m+k)!}{(m+2k)!}\right] \cdot \int_{-\pi}^{\pi}\int_{-\infty}^{\infty} g(y',\xi)\, e^{\mp im\xi}\, H_{m+2k}(\alpha y')\, dy'\,d\xi \tag{19}$$

Equation (17), with the coefficients of equation (19), represents the function g by a Gram-Charlier series in the radial direction, and by a Fourier series in the azimuthal direction.

Equation (14), with the coefficients of equation (19), becomes:

$$f(x,y) = \frac{\alpha}{\pi^{3/2}} \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \epsilon_m \frac{\left(k!\,(m+k)!\right)^{\frac{1}{2}}}{(m+2k)!}\, e^{-(\alpha^2x^2+\alpha^2y^2)}$$

$$\cdot \text{Real} \left[\int_{-\pi}^{\pi} \int_{-\infty}^{\infty} g(y',\xi)\, e^{-im\xi}\, H_{m+2k}(\alpha y')\, dy'\, d\xi \cdot U^{m}_{m+2k}(\alpha x, \alpha y) \right] \tag{20}$$

or, inserting equation (15):

$$f(x,y) = \left(\frac{\alpha}{\pi}\right)^2 \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \epsilon_m \frac{(-1)^k\, k!}{(m+2k)!} \left(\alpha^2x^2+\alpha^2y^2\right)^{\frac{m}{2}} L^{m}_{k}\left(\alpha^2x^2+\alpha^2y^2\right)$$

$$\cdot \left[B^{m}_{m+2k}(\alpha) \cos(m\phi) + D^{m}_{m+2k}(\alpha) \sin(m\phi) \right] e^{-(\alpha^2x^2+\alpha^2y^2)} \tag{21}$$

where:

$$B^{m}_{m+2k}(\alpha) = \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} g(y',\xi) \cos(m\xi)\, H_{m+2k}(\alpha y')\, dy'\, d\xi \tag{22}$$

$$D^{m}_{m+2k}(\alpha) = \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} g(y',\xi) \sin(m\xi)\, H_{m+2k}(\alpha y')\, dy'\, d\xi \tag{23}$$

Equations (21, (22) and (23) represent the fundamental inversion by Maldonado's method. Both analytical and numerical inversions were demonstrated in his series of three papers.

The method applies quite directly to density fields. The zero

value of the function $\Delta\rho$ studied in interferometry is arbitrary and can always be chosen such that the function is zero outside of a given circular boundary. Singularities do not occur in real density fields except for the spaces occupied by opaque objects. Cases with simple solid objects appear amenable to this method, but will not be discussed here.

b. The Numerical Procedure

Since experimental data are not obtained in analytical form, the indicated operations must be performed numerically. The contractual report resulting from the investigation by Olsen, et. al. [1966], included a computer program. However, it appeared more prudent to reprogram the basic equations to fit the available computer, and to more closely meet the needs of the particular application.

The form of equation (24) is a truncated form of equation (21):

$$f(x,y) = \left(\frac{\alpha}{\pi}\right)^2 \sum_{m=0}^{M} \sum_{k=0}^{K} \epsilon_m \frac{(-1)^k k!}{(m+2k)!} \left(\alpha^2x^2+\alpha^2y^2\right)^{\frac{m}{2}} L_k^m \left(\alpha^2x^2+\alpha^2y^2\right) \tag{24}$$

$$\cdot \left[B_{m+2k}^m(\alpha) \cos(m\phi) + D_{m+2k}^m(\alpha) \sin(m\phi)\right] e^{-\left(\alpha^2x^2+\alpha^2y^2\right)}$$

The numerical evaluations of the coefficients from equations (22) and (23) are accomplished approximately. Since the function is known to be zero outside of the circular domain of Figure (5), the integration over y' need only be made over finite limits, defined here as -S/2 to +S/2. The function g, which is always single-valued, is assumed to be constant within the discrete areas of the data plane represented

in Figures (6) and (7). This permits the integrals to be represented as a finite sum of integrals over the subregions:

$$B^m_{m+2k}(\alpha) = \sum_{i=1}^{IMAX}\sum_{j=1}^{JMAX} g(y'_j, \xi_i) \int_{\xi_{i-\frac{1}{2}}}^{\xi_{i+\frac{1}{2}}} \cos(m\xi)\, d\xi \int_{y'_{j-\frac{1}{2}}}^{y'_{j+\frac{1}{2}}} H_{m+2k}(\alpha y')\, dy' \quad (25)$$

$$D^m_{m+2k}(\alpha) \sum_{i=1}^{IMAX}\sum_{j=1}^{JMAX} g(y'_j, \xi_i) \int_{\xi_{i-\frac{1}{2}}}^{\xi_{i+\frac{1}{2}}} \sin(m\xi)\, d\xi \int_{y'_{j-\frac{1}{2}}}^{y'_{j+\frac{1}{2}}} H_{m+2k}(\alpha y')\, dy' \quad (26)$$

The integrals are then evaluated over the sub-intervals with $\xi_{\frac{1}{2}} \equiv -\pi$, $\xi_{IMAX+\frac{1}{2}} \equiv +\pi$, $y'_{\frac{1}{2}} \equiv -\frac{S}{2}$, and $y'_{JMAX+\frac{1}{2}} \equiv +\frac{S}{2}$

The integrals over ξ are elementary, and the integrals over y' are evaluated by use of the derivative formula for Hermite polynomials [Erdelyi, et. al., 1953]. The resulting general formulae for B and D are:

$$B^m_{m+2k}(\alpha) = \frac{1}{2\alpha(m+2k+1)} \sum_{i=1}^{IMAX}\sum_{j=1}^{JMAX} g(y'_j, \xi_i) \left[\frac{\sin(m\xi_{i+\frac{1}{2}}) - \sin(m\xi_{i-\frac{1}{2}})}{m}\right] \cdot \left[H_{m+2k+1}(\alpha y'_{j+\frac{1}{2}}) - H_{m+2k+1}(\alpha y'_{j-\frac{1}{2}})\right] \quad (27)$$

$$D^m_{m+2k}(\alpha) = \frac{-1}{2\alpha(m+2k+1)} \sum_{i=1}^{IMAX}\sum_{j=1}^{JMAX} g(y'_j, \xi_i) \left[\frac{\cos(m\xi_{i+\frac{1}{2}}) - \cos(m\xi_{i-\frac{1}{2}})}{m}\right] \cdot \left[H_{m+2k+1}(\alpha y'_{j+\frac{1}{2}}) - H_{m+2k+1}(\alpha y'_{j-\frac{1}{2}})\right] \quad (28)$$

which are valid for $m > 0$. The special case of $m = 0$ represents the axisymmetric solution. The terms within the brackets $[\ ]$ are evaluated by l'Hospital's rule: $D^0_{0+2k} = 0$, and

$$\lim_{m \to 0} \left[\frac{\sin(m\xi_{i+\frac{1}{2}}) - \sin(m\xi_{i-\frac{1}{2}})}{m} \right] = \xi_{i+\frac{1}{2}} - \xi_{i-\frac{1}{2}} = \Delta\xi \qquad (29)$$

c. The Computer Program - Holofer

In order to perform the inversions a FORTRAN IV computer program called HOLOFER, which includes a large number of variable parameters and optional modes of operation, was written. The complete program, with input parameter descriptions and the working data, is included in appendix C.

(1) The Basic Program. The data plane, called the G-array, consists of a discrete set of values of the function g of equation (11) taken over regular intervals of y' and ξ, of size IMAX by JMAX (Figure 7). The G-array is obtained in one of three modes. Mode one computes the G-array that corresponds to an optional test function by the numerical integration of equation (11) at regular intervals of y' and ξ. By generating its own data from a test function, mode one provides the basic program testing capability. In mode two operation, HOLOFER interpolates a regular G-array from an irregularly entered set of values such as those normally obtained in holographic interferometry. Mode three operation reads in a regular array of input values directly.

Next, equations (27) and (28) are evaluated for each set of coefficients B and D up to the series truncation values of m = MLIMIT

and k = KLIMIT. These B and D values are stored on external (DISK) storage.

Having computed the B and D coefficients, HOLOFER next evaluates equation (24) for each point in the test section individually. Either a cartesian or a cylindrically aligned array of spatial coordinates can be accommodated by choice of input parameters. In the evaluation of equation (24), truncation of each series is performed by first specifying an input parameter epsilon. If a certain number of terms in a row remain less than epsilon times the existing partial sum, the series is truncated. If the index values reach the limiting available set of B and D coefficients before the series has converged, the most recent few values of the partial sum are averaged.

(2) Add-On Functions. A second G-array may be computed and added to the original G-array prior to the inversion. The array is computed in the mode one operation from any given function. In addition, a gaussian random number of specified standard deviation may be added to the second G-array. The first feature provides the capability of adding a smoothing function to data with severe properties, such as from shock waves or opaque discontinuities in the interferograms. Peripheral information about the function may be used to determine a desirable function. The known add-on function is then automatically subtracted from the solution. The second feature allows one first to perform the inversion

of a set of data with a known error size and then to compare the result with a previous solution in order to estimate the possible error in the solution.

(3) Shock Waves. The polynomial series representation of discontinuous or high gradient regions such as those from shock waves requires a large number of terms. Accordingly, the inversion routine requires a large index of m and k to represent steep gradients. If sufficiently many data points are entered on input to provide adequate significance in the high order terms, one can utilize rather large values of MLIMIT and KLIMIT to obtain reasonably accurate inversions.

If one has prior knowledge of the function to be inverted, such as an isentropic solution of a flow field, an add-on function can be added which will smooth the net function at such discontinuities, i.e. the subtraction of a "plug" function shape in Figure (12). Such modification enhances the convergence of the resulting function, if it is well matched, and it therefore improves the accuracy of the result. An analogous method applicable to the Abel axisymmetric inversion has been described by Bennet, Carter, and Bergdolt [1952], called the method of "reduced functions," which operates on the fringe curve directly.

d. Symmetry

HOLOFER is designed to invert an asymmetric field. There

are, however, several simplifications to the process that result from symmetry in the field.

(1) Axisymmetric Fields. For axisymmetric fields, the angular variations disappear with the result that only one view or one line of the G-array need be considered. The coefficient evaluation integrals over the angular domain thus become zero, for all except the $m = 0$ case, and the summation process simplifies to a single series. Computation time in this mode is reasonably short, from 20 - 60 seconds on the IBM 360-67 digital computer for KLIMIT = 1000. The majority of program testing has been in this mode.

(2) With Planes of Symmetry. Planes of symmetry in the field reduces the angular requirements of the input data. In Figure (8), the data plane is shown divided into unique segments, only one of which need be supplied. Note that even in the asymmetric case, the data plane has two duplicate (though inverted) segments. The function looks the same from opposite sides of the domain, with reversed argument: $g(\gamma', \xi) = g(-\gamma', \xi + \pi)$. Additionally, the domain of orthogonality is reduced for cases of planar symmetry greater than one. The regions which are not cross hatched, are subdomains of orthogonality. The region of integration for coefficient evaluation is reduced accordingly in the program. The index value m in the series representation corresponds to the Fourier mode of the expansion of the function f in the azimuthal

direction. Therefore, for cases of planar symmetry the coefficients are zero for certain values of m . To summarize:

Asymmetric: uses all B and D coefficients:

One Plane: uses B coefficients only, all

Two Planes: uses B coefficients, m = 0, 2, 4, 6,

Three Planes: uses B coefficients, m= 0, 3, 6, 9 ....

Four Planes: uses B coefficients, m=0, 4, 8, 12 ....

Five Planes: uses B coefficients, m= 0, 5, 10, 15 .....

etc.

Axisymmetric corresponds to an infinite number of planes of symmetry: uses B coefficients m = 0 only.

While the higher numbers of symmetry planes have little practical value, they are included automatically in the generalization of the program symmetry tests.

3. Theoretical Results

a. Artificial Data Generation

Mode one of the program generates the G-array for a specified function. Several functions are included in subroutine FUNCT for use as program tests or as add-on functions. Provision is also made to enter an arbitrary function in subroutine SPFUN by simply entering the Fortran cards describing the function. In addition, a set of numerical values may be read in and used as a function with linearly

interpolated values between the given points. The G-array is evaluated at each value of y' and ξ as shown in Figure (7). Simple summation of the function value at 3.* (IMAX) points along the line y' = const. is used to evaluate the integral. Once the data plane is filled, the inversion process makes no distinction between computed or experimental data in the G-array.

b. Test Inversions

(1) Axisymmetric. Figure (9) represents the function, computed fringe curve, and inverted solution of a gaussian function similar to Maldonado's test case: $f = e^{-\beta^2(x^2+y^2)}$ (30)

The original analytical function is shown as a solid line, while the corresponding computed fringe curve is shown to a separate scale as a series of boxes. The greatest difference between the inverted solution and the original function was 0.001, or 0.1 percent of the maximum function value.

Figure (10) is a cosine-squared function:

$$f = \cos^2\left(2\pi\left(x^2+y^2\right)/3\right) \tag{31}$$

The greatest inversion error was 0.5 percent of the maximum functional value.

To determine the capability of the program to invert shock-wave type discontinuities, a circular square wave, or "plug" function, was inverted. Figure (11) shows a rounding off of the discontinuity that is

typical of a truncated series representation. The maximum error that occurs in the nearby region is 6.2 percent. At the center of the function, at radius zero, the series does not converge. The resulting 8.6 percent error is typical of the center point of inverted functions containing discontinuities.

Winckler, in an extensive application of the Abel inversion method to free jets [1948], used a hypothetical test function that is shown in Figure (12). Plots of the solutions he obtained are shown for comparison of the two methods. The characteristic "overshoot" of the Abel method near the discontinuity is shown, where the largest error by the Maldonado inversion was 3.8 percent.

Figures (13) and (14) are inversions of typical axisymmetric density functions from the Winckler analysis of a free jet. They demonstrate the capability of the inversion routine with realistic types of functions. Both inversions were accurate to within 2.6 percent.

(2) Non-Axisymmetric. Figure (15) represents three cross-sections of the same gaussian function as before, now centered at $x = .1$, $y = 0.$ cm. This corresponds to the test example that Maldonado used in the verification of his inversion scheme. The solution is everywhere accurate to within 0.8 percent with a very low number of series terms. The function is planar symmetric and thus tests only the cosine terms of the expansion.

Figure (16) represents three-cross-sections of an asymetric test case. The function is an elliptical cone of base diameters 0.7 x 0.5 cm., centered at x = 0.707, y = 0.707 cm. In the solution, the tip of the cone is rounded by the natural smoothing characteristics of the inversion method, but otherwise the maximum error is 1.5 percent. This test case represents a complete test of the inversion procedure, utilizing both B and D terms of both series expansions.

III. EXPERIMENTAL APPROACH

A. DESCRIPTION OF THE APPARATUS

1. The Holographic Table

A holographic work table was designed and constructed to provide optical bench facilities about a free jet, as shown in Figures (17) and (18). The table was designed to evaluate various arrangements of optical components to achieve a wide angle of holographic field of view, and to suppress the vibration caused by the free jet noise.

The table was made of pressed plywood laminate, two and one-half inches thick. The four by six foot table was mounted on a rotatable set of plywood boards, with a centerhole of eight inches diameter to accommodate the free jet. This arrangement provided the capability of rotating the entire table about the flow field. Below the rotating arrangement, the table rested on four small tire tubes to provide

structural vibration isolation from the building. For experiments conducted in C-W gas laser holography with up to thirty second exposure times (without the free jet) the arrangement was very successful. Beneath the inner-tube mounting, the table rested on four standard automobile type screw jacks. Recessed jack points allowed the table to be readily tilted to about 15^{o} about the flow field.

A Korad K-1 pulsed ruby laser operating at a wavelength of 6941. A. was employed with a Pockels cell Q switching device. The resultant effective hologram exposure time was about 20 ns., eliminating problems with vibration during the hologram exposure. There does remain the problem of vibratory misalignment of the optical components between the two exposures of the holographic interferogram. To help damp accoustically-caused vibration, the mirrors were all mounted on heavy metal blocks. The weakest link in the setup appeared to be the beam splitter holders. They were lightest of the table components, and as a result of their vibration, holographic interferograms obtained tended to have a finite fringe. Occasionally a hologram would be unusable because the fringe spacing became too fine to resolve.

2. The Test Section

The table was mounted around a standing free jet. The plenum chamber was about 45 cm. long by 30 cm. diameter. The jet extended 45 cm. above the plenum chamber with an inside diameter of 3.18 cm. and

a throat diameter of 2.0 cm. at the exit. The test section was defined by a square plexiglass enclosure, the four inside surfaces of which were inscribed with a one centimeter square grid. The grid box provided some vibration insulation for the jet, and also produced a self-contained coordinate system for the hologram and the corresponding photographs. The holograms were arranged about the grid box as shown in Figure (19), with the coordinate system established as shown. Commercial ground glass plate was used as the diffuser.

3. Laboratory Techniques

Alignment of the laser beam with the optical components was accomplished by aligning a continuous wave helium-neon gas laser through the rear mirror and along the ruby axis of the pulsed laser. A -20 cm. meniscus lens of inexpensive quality was used to diverge the beam. Care must be exercised in this arrangement to insure the convex side of the lens is towards the ruby laser. Concave surfaces in the near vicinity of a high power laser can themselves focus a high density of reflected energy, which might damage the laser itself or any near components.

Relative intensities of reference to object beams of about 4:1 were found to yield good holograms.

Since the arrangement of the several mirrors, beamsplitters, etc., so as to have the same optical path length from source to hologram for each beam is a tedious procedure, a cork pinboard was designed to

facilitate the table arrangements. Threads of the same length, each one representing a laser beam, were stretched from the laser source to their various holograms via different routes over a sketch of the table and held in position with pins. The table has a six inch grid painted on its surface to facilitate location of the components from the grid on the sketch. The pin board saved many hours of arrangement time.

Holograms were made on Agfa-Gaefert 8E75 holographic plates. Development was five minutes in Kodak D-19 developer, 30 seconds in acetic acid stop bath of standard dilution, five minutes in standard fixer, one minute water wash, followed by immersion in wetting agent (Kodak PhotoFlo) prior to drying. It was observed that good reconstructions of restricted field of view could be obtained immediately after developing, while the plate was still wet. Normal reconstructions were made with the continuous wave gas laser at 6328A. The resulting image magnification from reconstruction at a different wavelength was not considered deleterious. The technique used in making photographs for data extraction will be discussed in the next section.

B. HOLOGRAPHIC EXPERIMENTAL RESULTS

1. Non-Flow Holographic Experiments

Holographic experience was gained with a Spectra-Physics model 124 gas laser. Both back-lighted scenes and diffuse objects were recorded.

Real time interferometry of a test area was demonstrated and surface deformation interferometry was obtained. Figure (20) shows the interference pattern on the surface of a small clamping device under strain. Figure (21) is one view of the interference pattern about a cigarette. Between exposures of the hologram, the cigarette had been radiantly heating the platform below it for about three minutes, and the platform shows the interference pattern caused by the resulting thermal expansion. The gas laser was used to confirm the optical arrangement of Figure (19) for multiple hologram viewing of the free jet test scene. Successful holographic interferograms were obtained of cigarettes at all hologram positions.

2. Free Jet Experimentation

The Korad giant pulse laser was installed on the table and holograms taken of the free jet exhausting to the atmosphere. Figure (22) shows a shadowgram taken at 35 psig plenum pressure. A shadowgram is produced directly on the hologram plate when the holographic image is recorded by a single exposure of the dark field technique (with the ground-glass diffuser absent).

Figures (23) and (24) show two views obtained from the same holographic interferogram of jet flow at 60 psig. The interferometric data from this hologram were inverted to provide an axisymmetric solution. The results of the solution follow in section D. Holographic

interferograms were also taken at 25, 40, 45, and 50 psig., but were not reduced. Turbulent flow perturbations became more predominant as the pressure was reduced.

To provide a non-axisymmetric test of the method, the table was tilted 11^{o} clockwise about the y axis of the table. As a result of the tilt, horizontal cross sections through the field became planar symmetric about the x-z plane. The solution of planar symmetric fields require a 90^{o} field of view. Three simultaneous holographic interferograms were taken about the tilted jet at 60 psig. with the arrangement shown in Figure (19). Each of the three holograms provided a field of view of about 15^{o}, one of which had several degrees obscured by the corner of the box. To provide more complete coverage, the table was rotated and additional holograms taken. Two rotations were required. Figures (25) and (26) show the interferograms taken at 5^{o} and 85^{o} which were used in the data reduction.

3. Experimental Techniques and Considerations

Previous work at this laboratory [Sullivan, 1968] had shown the intensity transmission of collimated laser light beyond commercial ground glass falls below 30 percent of the incident intensity beyond viewing angles of about $\pm 8^{o}$. Since the diffraction capability of a holographic plate exceeds $\pm 8^{o}$, the ground glass represents the limiting factor to the field of view. In fact, usable holographic interferograms were obtainable with from $\pm 5^{o}$ to $\pm 10^{o}$ field of view, centered about the object beam direction,

the field size being a function of the ratio of the intensities of object beam and reference beam.

The holographic interferogram appears as a completely three-dimensional set of fringes to the observer. Only special cases of the fringe pattern can be localized, those corresponding to Young's fringes (the finite fringe background pattern), or those corresponding to regions of spherical or cylindrical density variations. To obtain usable data from the fringe number function, one must sample discrete segments of the information. This is done by recording a series of photographic interferograms at regular angles about the test section. For a completely general field one requires 180° field of view, while the planar symmetric case studied required 90°.

There are two basic methods of obtaining sufficient angular coverage of the field. The first is to take several individual holograms, rotating the relative angle between the hologram setup and the test section for each hologram. Unsteadiness in the flow between the exposures will introduce errors. The second method involves arranging a series of holograms about the test section for simultaneous exposure. Intervals in the data from the second method are filled by interpolation. Interpolation over large angles requires that the function vary slowly in the angular direction. This experiment utilized a combination of both methods.

a. Photographic Technique

A normal photograph of the hologram records an image of the focus plane as shown in Figure (28). Each position on the photograph represents the line of sight from the image to the aperture (Figure 27). All of these lines of sight will represent a non-parallel set of lines. For reasonable camera focus distances, the deviation from parallelism is small and may be neglected. The spatial filtering technique shown in Figure (29) allows the selection of parallel sets of lines of sight for the recorded fringe pattern. The aperture stop at the focal plane of the lens filters out all but the lines parallel to the central angle. The resulting photographs are simpler to analyze since the angles are constant. In addition, fringe data from any z plane may be obtained from the single photograph. Mach-Zehnder interferograms, because of their single collimated source, provide the same type of interferogram.

The technique utilized for this investigation was an application of the lensless focusing capability of the hologram and is easier to achieve then the previous two methods. Figure (29a) shows a hologram being reilluminated by a conjugate reference beam of small diameter. The real image of the test scene is formed in the same position as shown in Figure (3). Because the reconstruction beam is of small size, the illuminated portion of the hologram represents a small aperture . The resulting image has a large depth of field and a photographic film placed at the

image records the test scene. Additionally the rays may be focused in the most desirable plane by positioning the film plane, usually near the center of the disturbance. The lines of sight recorded represent the diverging bundle passing through the aperture position from the diffuser. The maximum angle of divergence at the edge of the test field encountered was $\pm 5^{o}$. For resolution greater than $\pm 5^{o}$, one must compensate for the variation. A subroutine of the computer program was written to accomplish this compensation, but the errors introduced by neglecting the bundle divergence have been acceptably small, and use of the routine has not been required.

The specular interference of coherent light from a diffuse surface causes a film graininess that is inversely proportional to recording aperature size [Tanner, 1967]. Although recording apertures of one millimeter were used, the speckle created no problems.

b. Data Reduction

To obtain the photographic interferograms, a camera back with viewing screen was placed in the position of the real image of the hologram as previously described (Figure 29a). The illumination of each position on the hologram corresponds to a particular effective aperture position. The hologram was positioned such that the desired set of front and rear grid lines lined up on the camera back viewing screen corresponding to the desired elevation and angle of view for the picture. Reference

to Figures (23) and (24) show grid alignments at $z = 3.$ cm., $\xi = 0^{o}$ and $z = 1.$ cm., $\xi = 0^{o}$ respectively. Polaroid P/N 55 film was used to record the image.

To obtain the graphical plot of the fringe number function, the negative was used in a photo enlarger to image directly upon graph paper. With the enlargement adjusted to match the scale of the paper, the positions of the maxima and minima of the fringes along the desired cross section were determined visually. The fringes were counted, and given a graphical elevation according to their number. Initial fringe numbering was arbitrary, commencing with a fringe well to one side of the field and proceeding across the field, the light regions representing integer values of the fringe number.

Figure (30) shows a typical working graph that was used for the $\xi = 5^{o}$ view of the tilted jet. The finite fringe reference line was drawn from the field edges and the curve was transcribed. Numerical values were then read from the curve at regular increments of radius (y') corresponding to the G-array size IMAX.

For the axisymmetric case the entire procedure was repeated for each level z plane to be solved. Since each aperture position on the hologram corresponds to a particular elevation and angle of view, a new picture was made for each horizontal z plane through the field.

For the asymmetric case, a new picture was taken for each angle ξ desired. When the angle desired was not available because of gaps in the holographic coverage, the curve was graphically interpolated from nearby curves on either side. Fringe curves were obtained in the standard manner for angles on each side. The two curves were then overlayed on a light table and an intermediate curve drawn at the proper relative distance.

IV. THE APPLICATION OF THE INTEGRAL INVERSION TECHNIQUE TO THE EXPERIMENTAL RESULTS

A. AXISYMMETRIC SOLUTION

The interferometric data from the hologram of the free jet at 60 psig. were reduced at eight positions out to z = 2.0 cm. in the manner described previously. Figure (31) shows the fringe curves obtained at z = 0.5 cm. and z = 1.5 cm. with their corresponding density solutions. The complete set of solutions are shown topologically in Figure (32). The solution compares quite well with the topological features of the free jet solutions obtained by Ladenberg, Van Voorhis, and Winckler [1949] in their very extensive interferometric analysis of free jets of one centimeter diameter. Figure (33) is an isodensity line plot of the obtained data. Qualitatively, the densities compare very well with the similar plot of the Winckler solution (Figure 34). Exhaust density in the central region

of the exit plane agrees within 0.2 mg/cc (5.3 percent of the maximum field density) and in the central region at z/d=1. (d= nozzle diameter) within about 0.1 mg/cc (2.8 %), rising to a maximum of 0.4 gm/cc (10.6%) between. At a relative radius of one-half, the difference in the two solutions runs from about 0.05 mg/cc (1.4%) at z/d=0. rising to about 0.2 mg/cc at z/d=0.25 and falling to 0.1 mg/cc at z/d=1. At the jet radius, the maximum differences fall to about 0.05 mg/cc.

B. NON-AXISYMMETRIC SOLUTION

Figure (35) shows the measured fringe curves obtained for the free jet at 60 psig. and eleven degrees of tilt, on a horizontal plane which intersects the jet axis at a point 0.5 cm from the nozzle. A total of nine angular positions were sampled, every other one being shown in the figure. The significant trend in these data is the shoulder increasing with decreasing angle on the left-hand side and with increasing angle on the right-hand side. A contour map of the data surface from $\S = 0^{o}$ to $\S = 90^{o}$ is shown in Figure (36). Since not all data could be obtained simultaneously, the run number from which the holograms used were obtained is shown. The non-regularities appear to be the effects of errors introduced in the correlation of angular views taken at different times. A data smoothing technique used by Maldonado, et. al. on this data plane was that of fitting smooth curves to the plotted points and adjusting the data to fit the

smoothed curves. For this experiment, however, the data were used without smoothing. Figure (37) shows a sketch of the tilted plane through the jet, with the five lines shown along which solutions were obtained. Figure (38) shows the solution at the five diametric cross sections from $\xi = 0^{\circ}$ to $\xi = 90^{\circ}$. The effects of the shoulder variation in the input data show up here as variations in the position and size of the density "valley" which one can see in the axisymmetric topological plot of Figure (32). The comparison of the solution from the tilted plane at $\xi = 0^{\circ}$, 45, and 90° is made with the solutions taken from the axisymmetric experiment in common plots of the two functions. The shoulder and valley features of the two solutions appear consistent. The central "hill," is consistent in the two solutions, to the point of showing a common inflection in the slope near radius $\pm$ 0.5. The outer five points on either side of the 90° curve include convergence errors arising from failure of their series evaluations to converge. The maximum difference in the two solutions is about 8 percent, although the mean deviation is much less.

C. DISCUSSION OF THE ERRORS

1. Numerical Inversion Errors

Several function shapes were investigated with the axisymmetric inversion to determine the effect of inversion parameters upon the accuracy of the inversion. Errors resulted from two basic causes: one

was the failure of the series evaluation to converge within the maximum number of terms for which computed coefficients B and D were available; the other was the approximation caused by representation of the continuous function g by a discrete set of values.

Convergence of the series evaluation is fastest in regions where the functional shape matches a gaussian. As few as five terms of the K series were sufficient to evaluate the gaussian function to within one percent everywhere. The asymmetric test of Figure (15) of the displaced gaussian used only a maximum of 25 terms, corresponding to MLIMIT=5, KLIMIT=5 to achieve accuracy to within 0.8 percent. The opposite extreme occurs when functions with steep gradients or discontinuities are inverted. For example, in the test of Winckler's function (Figure 12), the region near the discontinuity required up to 1700 terms for convergence; at the discontinuity itself, convergence was not achieved with 2000 terms. For functions with such discontinuities, or for experimental data inversions where there are normal irregularities in the data, the center position (at radius zero) often fails to converge, resulting in error at that position of up to 10 percent. When convergence has failed, however, the index count of the output information clearly indicates the failure, and this point value may be simply neglected. Interpolation of the missing value from nearby convergent points normally reduces the error to within two percent.

Decreasing the number of points of the data plane reduces the significance of the higher order modes of the series expansion. This rounds normally smooth functions, and causes oscillations in functions with discontinuities. For example, the step function of Figure (11) had a maximum error of 6.2 percent with IMAX = 200, and maximum error of 12.7 percent with IMAX = 100. Additionally, the inversion at IMAX = 100 caused a severe oscillation over most of the step.

Since there is no mathematical requirement in the inversion for regular spacing of the data grid, modification of the numerical equations could be made to accommodate finer mesh sizes in the regions of special interest.

2. Errors in the Data

The errors in the final solution are mostly due to errors in the data. There are several independent sources of data errors, covered here individually.

Probably the greatest source of error in the data arises from the unsteadiness of the jet flow. The three apparent perturbations in the solution of Figure (33) correspond to 0.1 mg/cc and 0.2 mg/cc respectively. Based upon the maximum density in the field, these correspond to 2.8 and 5.6 percent errors. Variations in the flow between the runs are considered responsible for the major fluctuations evident on the data plane shown in Figure (36). The asymmetric solution tends to spread

errors over the whole field, reducing their effect by statistical averaging.

Graphical positioning of the interferogram and scale matching in the photo-enlarger to graph paper step is accurate to within two percent. However, a two percent position error might be magnified to a five percent density error in regions of normally high density gradient. The accuracy to which one can determine the fringe number position in reading the interferogram is within $\pm$ 1/8th. of a fringe, or within about 1.5 percent of the maximum fringe number of the flow interferograms studied. Winckler has provided an analysis of the relative merits in reading several different fringe arrangements to minimize the fringe number error. Figure (39) shows the inverted solution of the axisymmetric test case at z = 0.75 cm. with a second solution superimposed. The second solution has been made from data with added random error of 0.0625 standard deviation, corresponding to $\pm$ 1/8th. fringe number error in reading the data. The resulting error varies with a maximum of 2.1 percent.

The background finite fringe spacing is assumed to be constant through the field. Figure (40) illustrates the type of error that could exist by the failure of linearity. An error curve that starts at zero at the boundaries of the flow and rises to one-half fringe in the number

would yield a maximum solution error of 0.18 mg/cc at the center, or about 5.0 percent for the axisymmetric solution.

It is estimated in the solution presented herein that probable errors are less than 8 percent everywhere, corresponding to the maximum difference in comparison of the two solutions in Figure (38).

V. SUMMARY

A. CONCLUSIONS

A self-consistent method has been demonstrated for the acquisition of interferometric data about a three-dimensional density field of one plane of symmetry. The data set has been successfully inverted by the application of a recently develcped mathematical inversion scheme and shown to be reliable to within eight percent of the density range. The mathematical model has been tested for realistic density patterns, representing supersonic wakes and jets. The pulsed laser method of holographic interferometry can be successfully applied in environments relatively hostile to normal interferometry. There are no inherent restrictions to the application of the method to generally asymmetric fields. The optical arrangement of the system is highly flexible and can be modified for interferometric studies of wakes, rockets, turbomachinery flows and other hostile, or highly transient events.

B. RECOMMENDATIONS

Most of the errors introduced in the solution were the results of the data-reduction methods used. The use of an appropriately designed densitometer for determining fringe locations on the negatives might improve the accuracy of fringe location and merits investigation. Measurable features on some object in the test area should be incorporated for accurate scaling and positioning of the data coordinates.

Experimental improvements could be accomplished as follows. An effort should be made to increase the field of view from the diffuser plate to provide increased angular coverage. The use of heavy, naturally damped mounts should be provided for the beam splitters for better control of the spacing of finite background fringes.

C. ACKNOWLEDGEMENTS

The writer wishes to gratefully acknowledge Dr. D. J. Collins for his sponsorship, guidance and assistance during the course of this investigation; Dr. A. E. Fuhs for his initial encouragement in the field; Drs. R. L. Kelly, T. H. Gawain, L. V. Schmidt, and G. D. Sackman for their time and support during the period; Dr. A. B. Witte, of TRW Systems, Inc., for his initial mention of Maldonado's work; the technical staff of the Department of Aeronautics under R. Besel and T. Dunton, particularly N. Leckenby and G. Gulbranson, for their able assistance; B. Taylor, of

the Department of Physics, for the loan of many little lenses and mirrors; Mrs. Kathryn Todd, for her considerable effort in the final preparation; and my wife and family for their noble patience and encouragement.

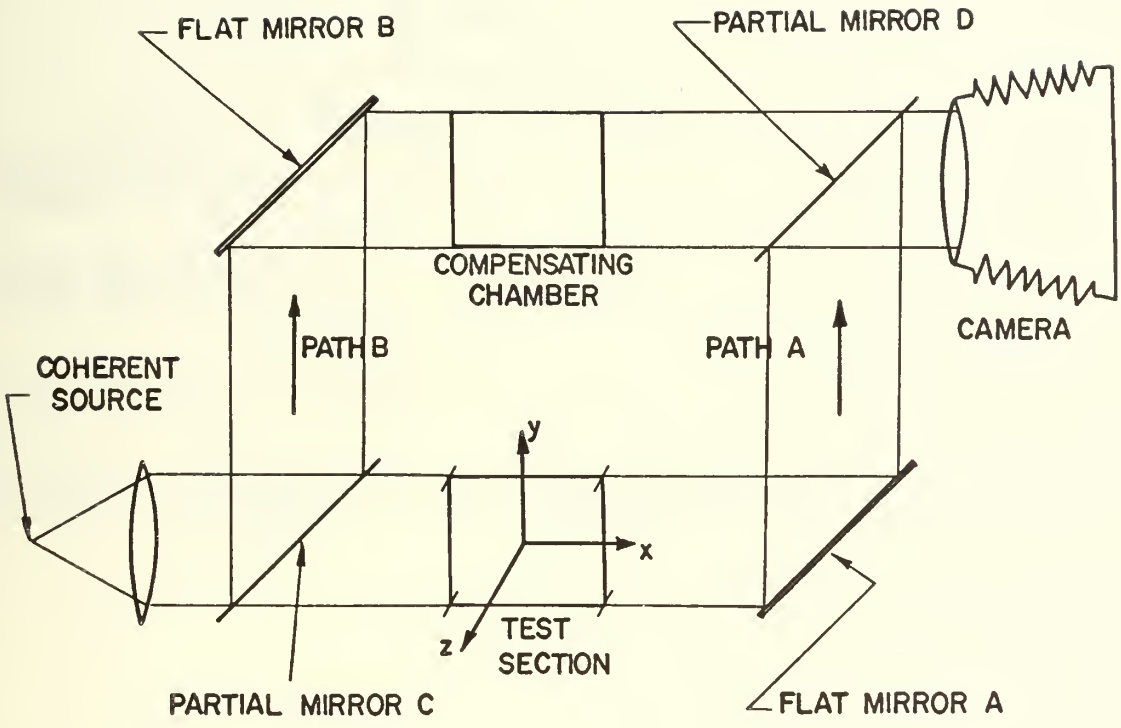

FIGURE 1: THE SCHEMATIC ARRANGEMENT OF A MACH ZEHNDER INTERFEROMETER.

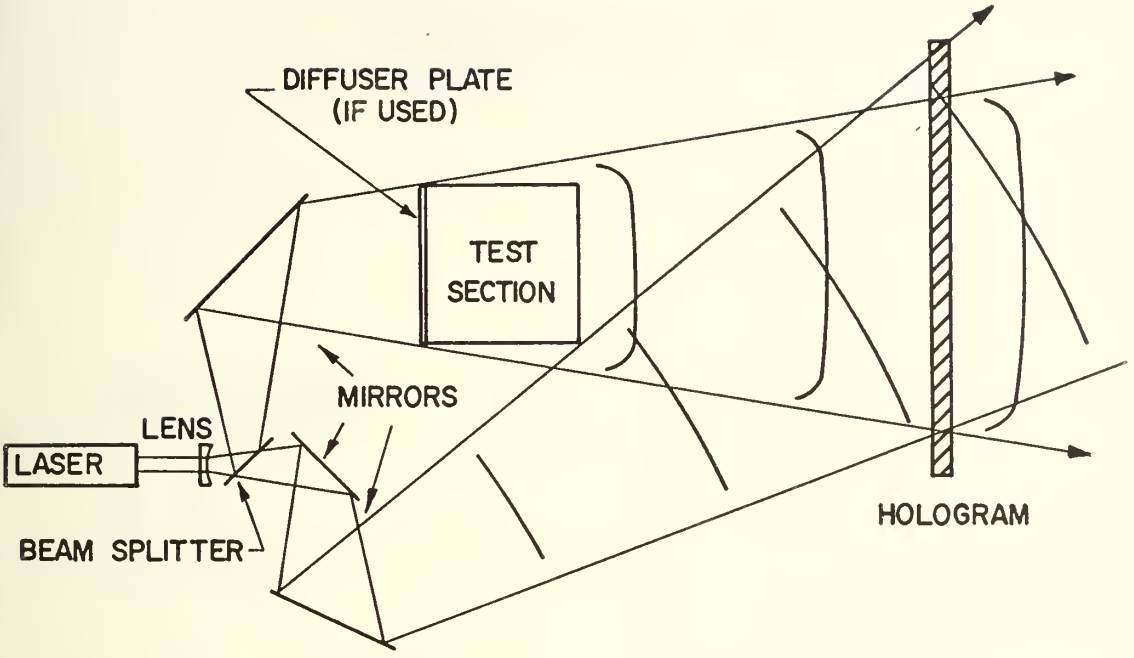

FIGURE 2: PRODUCTION OF A HOLOGRAM OF A TRANSILLUMINATED OBJECT.

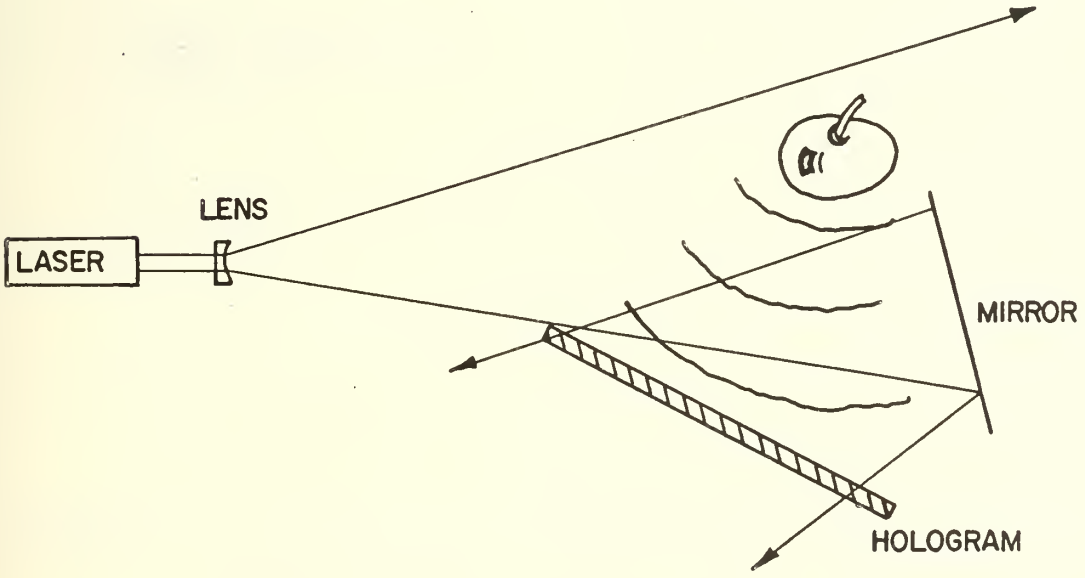

FIGURE 2a: PRODUCTION OF A HOLOGRAM OF A DIFFUSELY REFLECTING OBJECT.

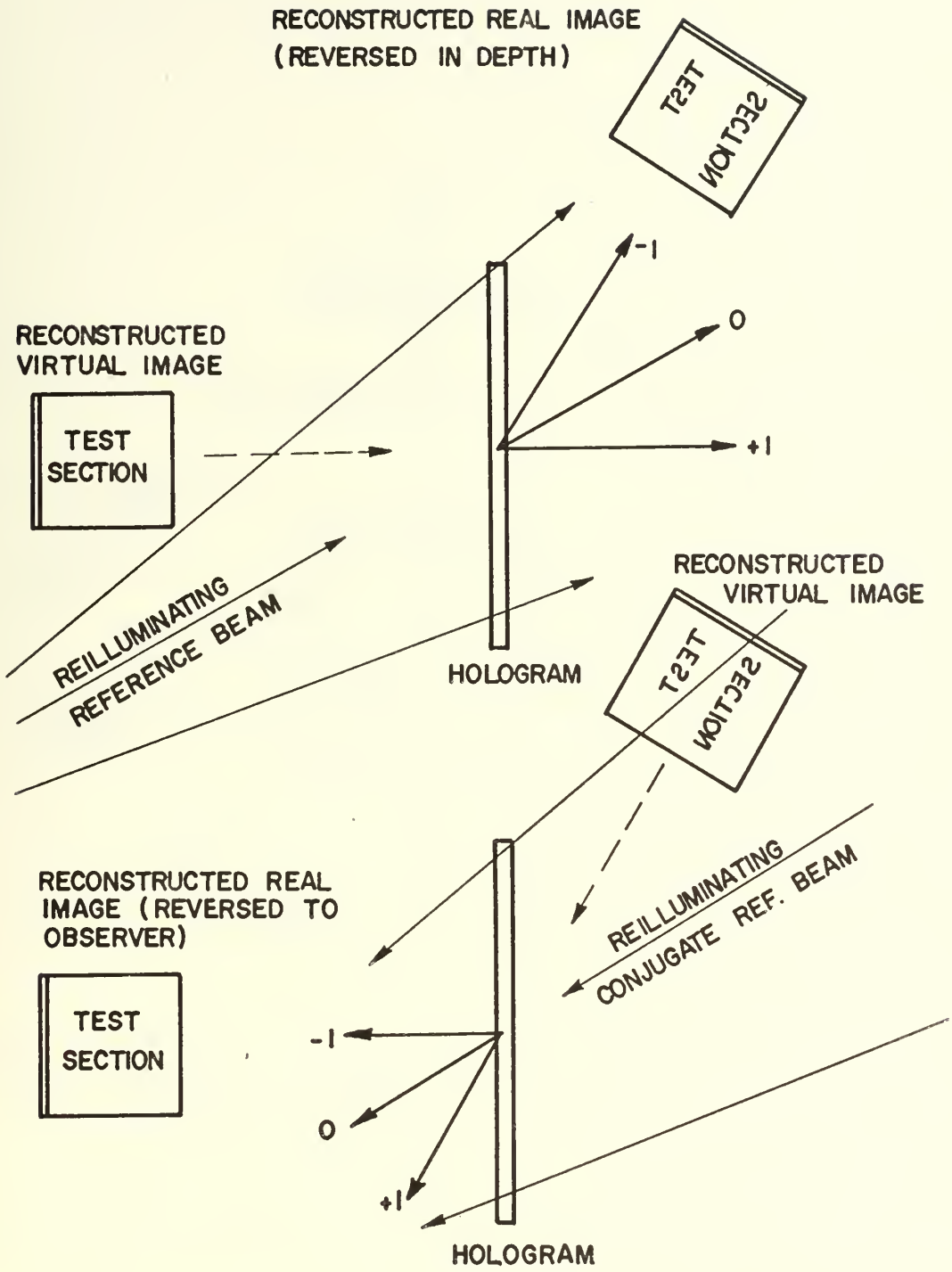

FIGURE 3: SHOWING IMAGES CREATED BY RECONSTRUCTION FROM EITHER SIDE.

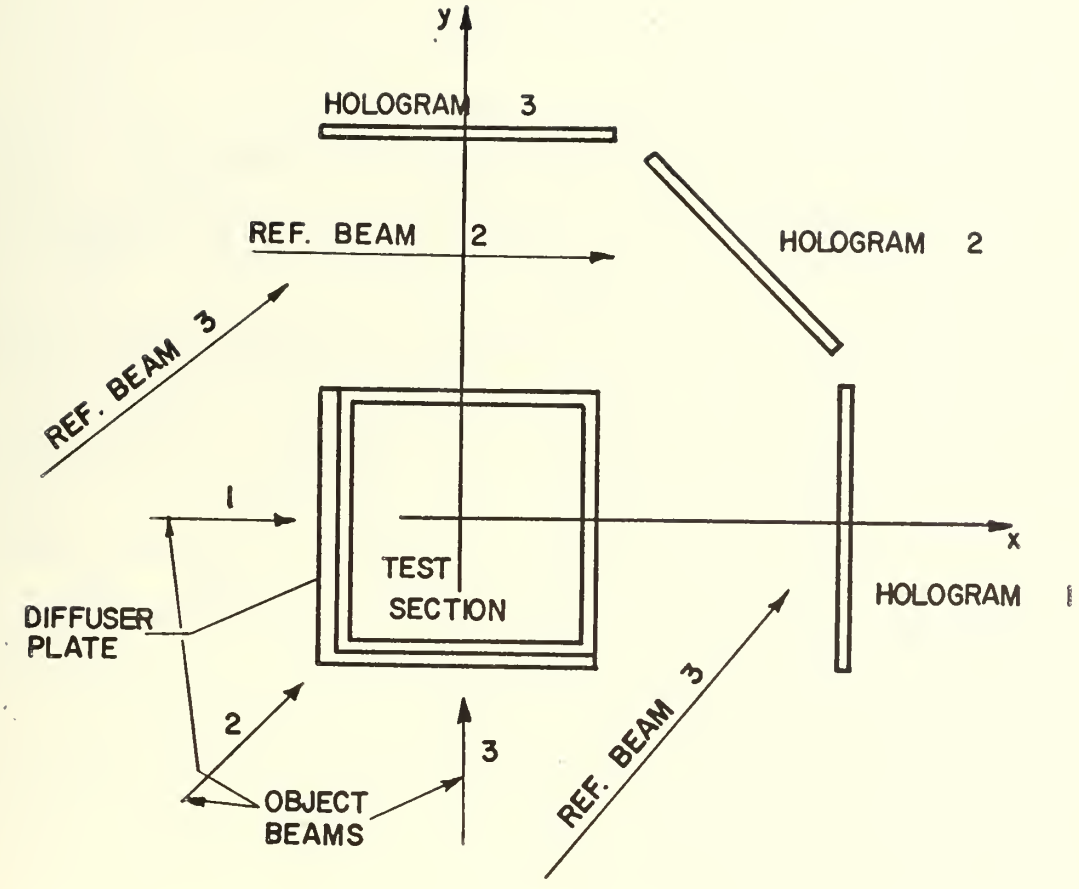

FIGURE 4: HOLOGRAM ARRANGEMENT FOR WIDE ANGULAR RANGE OF VIEWS.

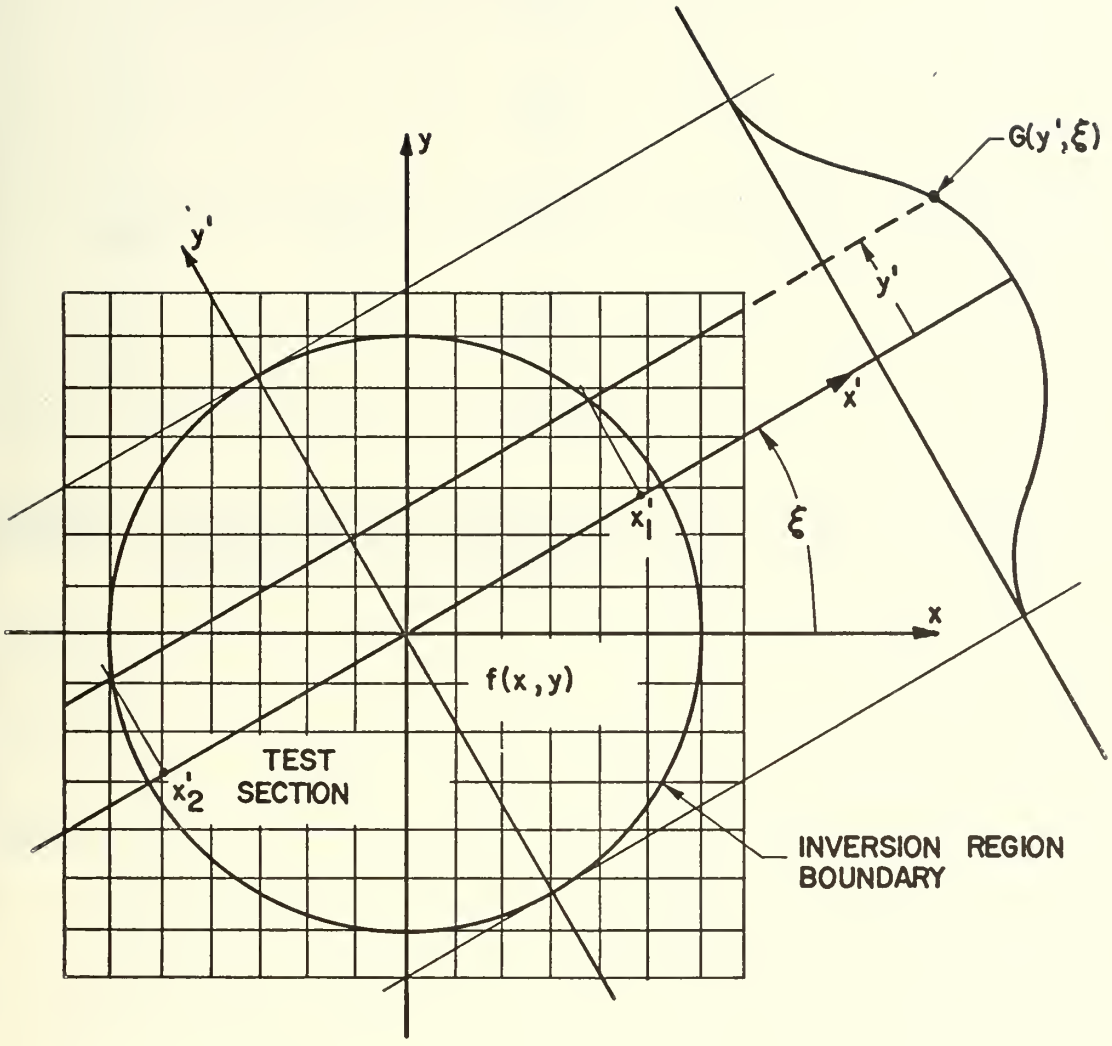

FIGURE 5: BASIC COORDINATE SYSTEM OF INVERSION PROCESS.

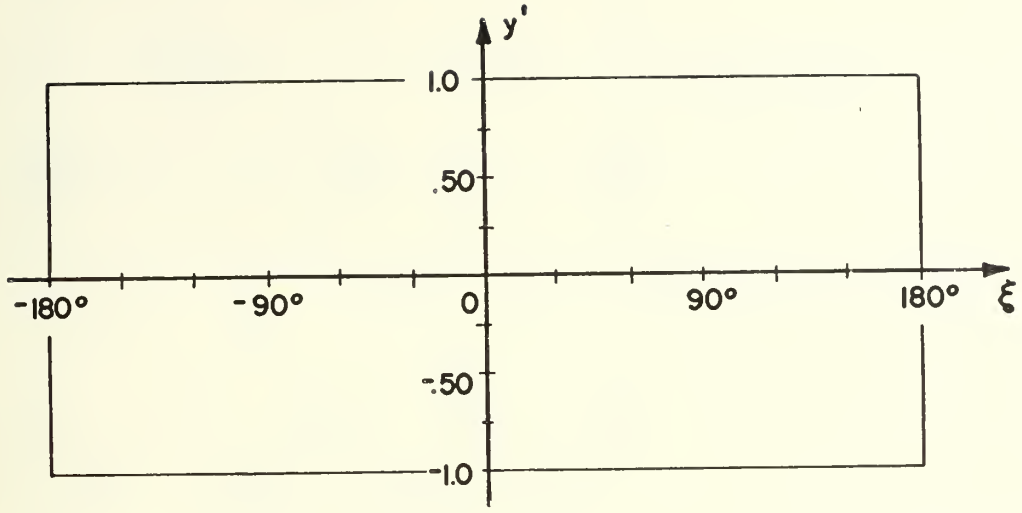

FIGURE 6: DOMAIN OF THE FUNCTION G OVER WHICH ORTHOGONAL INVERSION IS PERFORMED IN THE GENERAL CASE:

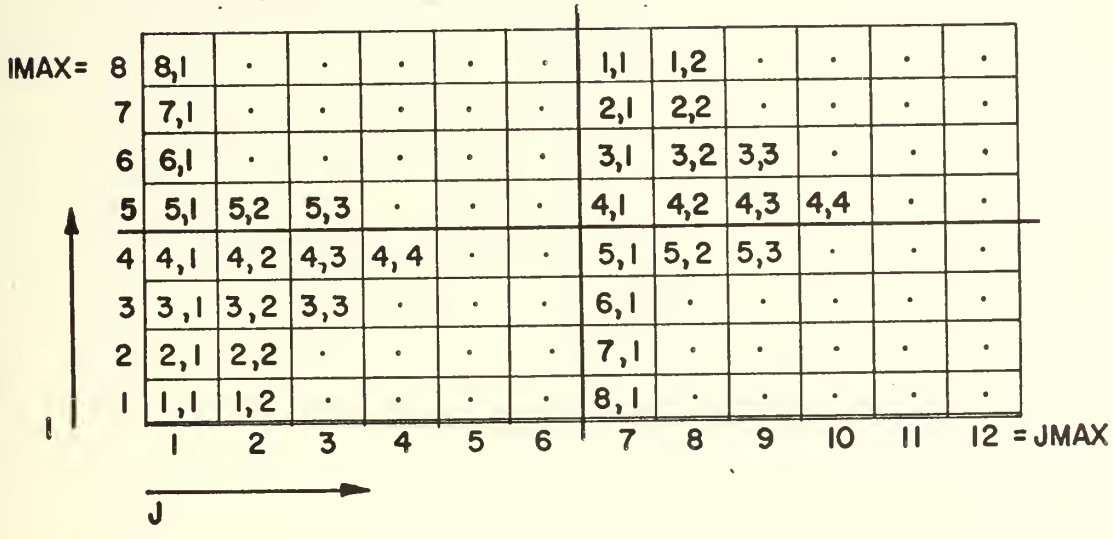

FIGURE 7: DISCRETE ARRAY OF G VALUES CORRESPONDING TO DOMAIN OF FIGURE 6. SHOWING DUPLICATION OF THE FUNCTION.

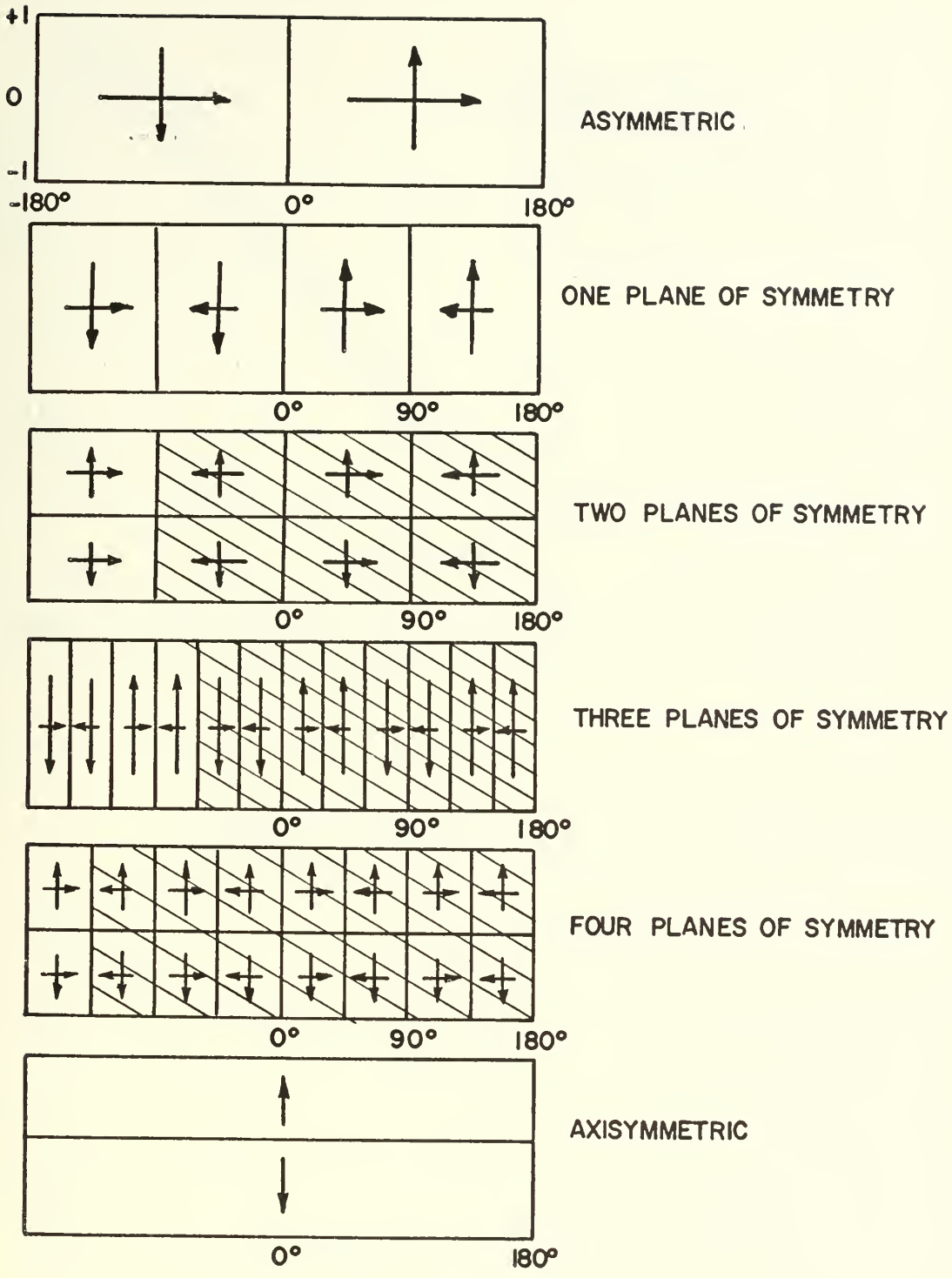

FIGURE 8. THE EFFECTS OF SYMMETRY OF THE FUNCTION F UPON THE REPEATABLE CHARACTER AND INTERVAL OF ORTHOGONALITY IN THE G FUNCTION.

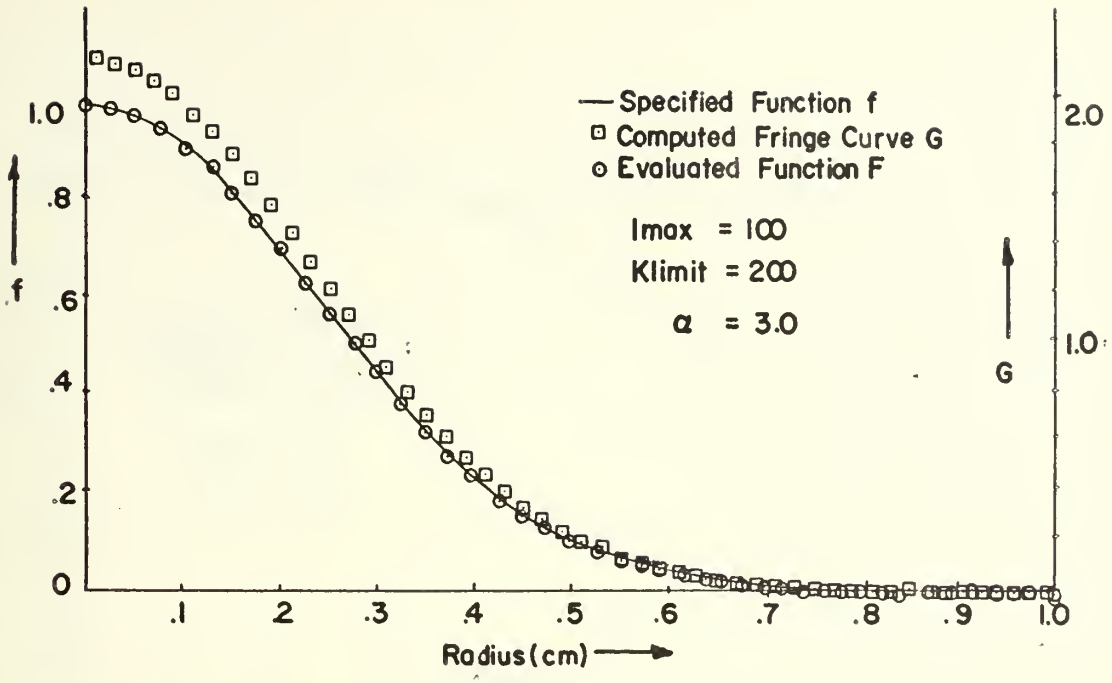

FIGURE 9. AXISYMMETRIC TEST CASE — GAUSSIAN FUNCTION

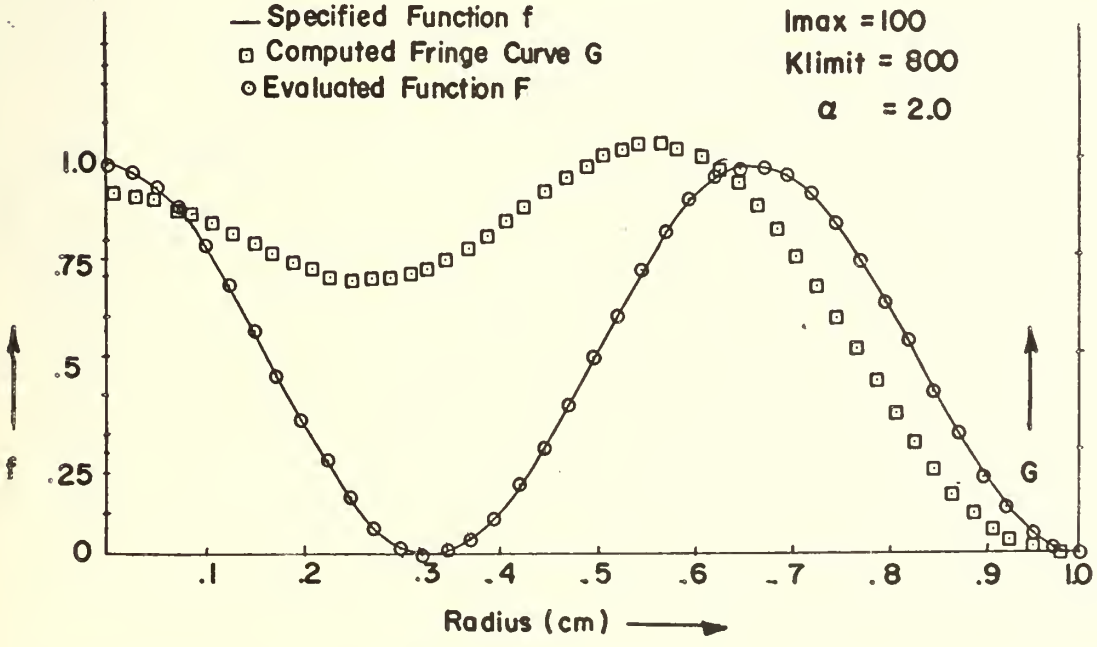

FIGURE 10. AXISYMMETRIC TEST CASE — $(\text{COSINE})^2$ FUNCTION

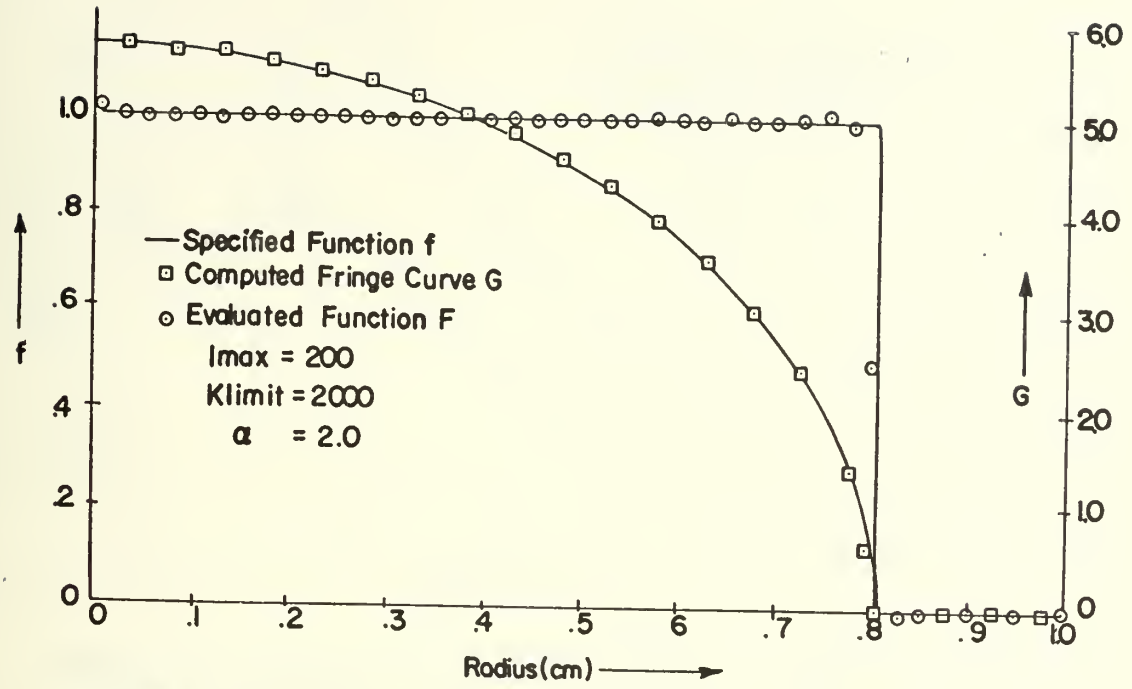

FIGURE 11. AXISYMMETRIC TEST CASE — STEP FUNCTION

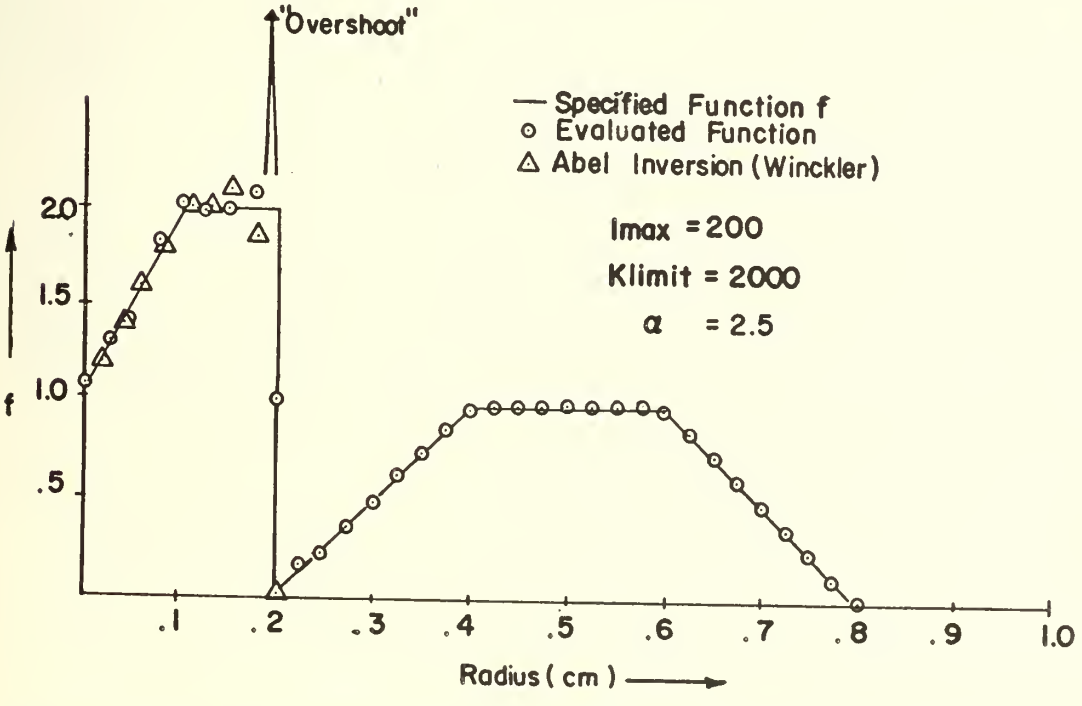

FIGURE 12. AXISYMMETRIC TEST CASE—WINCKLER'S TEST

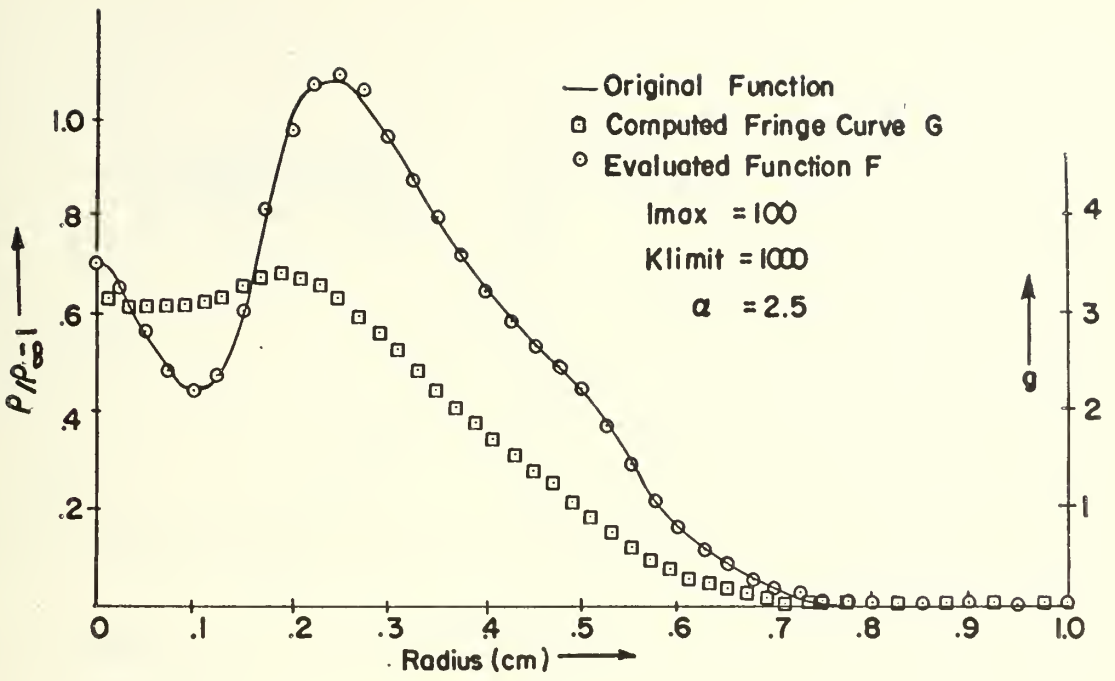

FIGURE 13. AXISYMMETRIC TEST CASE — FREE JET FLOW #1

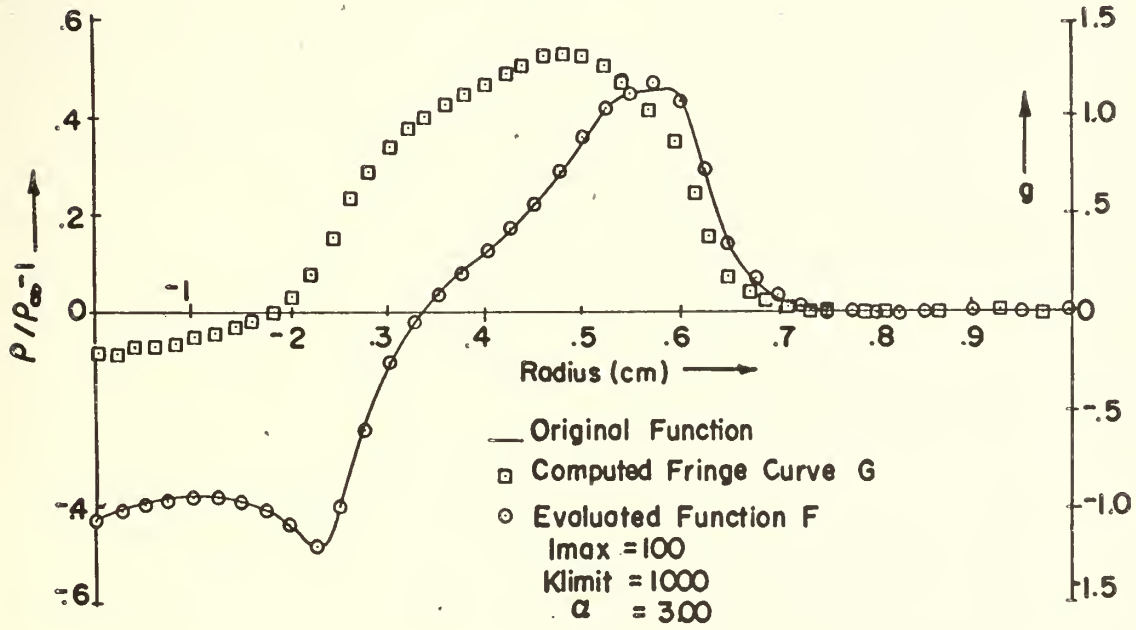

FIGURE 14. AXISYMMETRIC TEST CASE — FREE JET FLOW #2

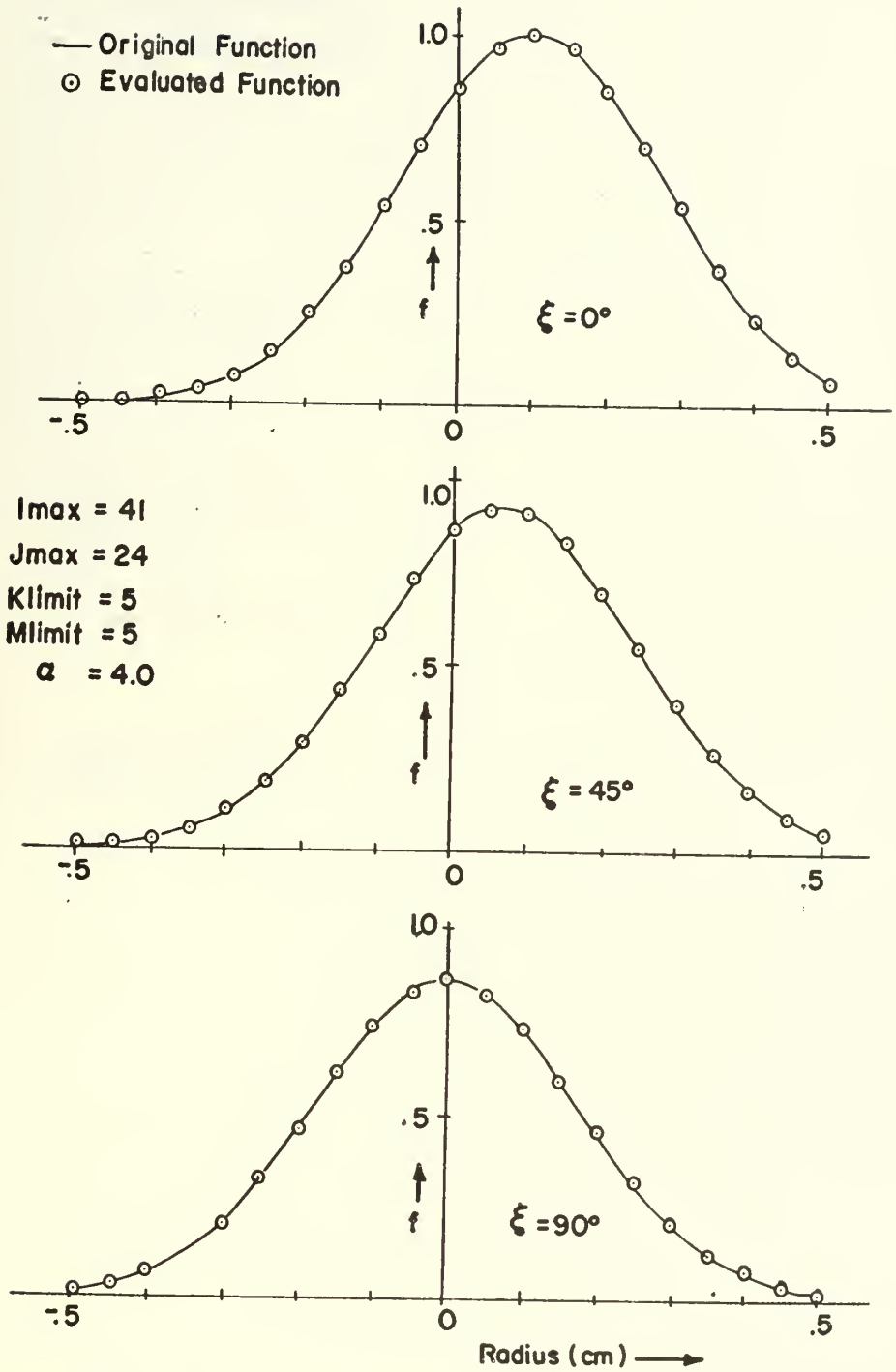

FIGURE 15. TEST INVERSION — DISPLACED GAUSSIAN

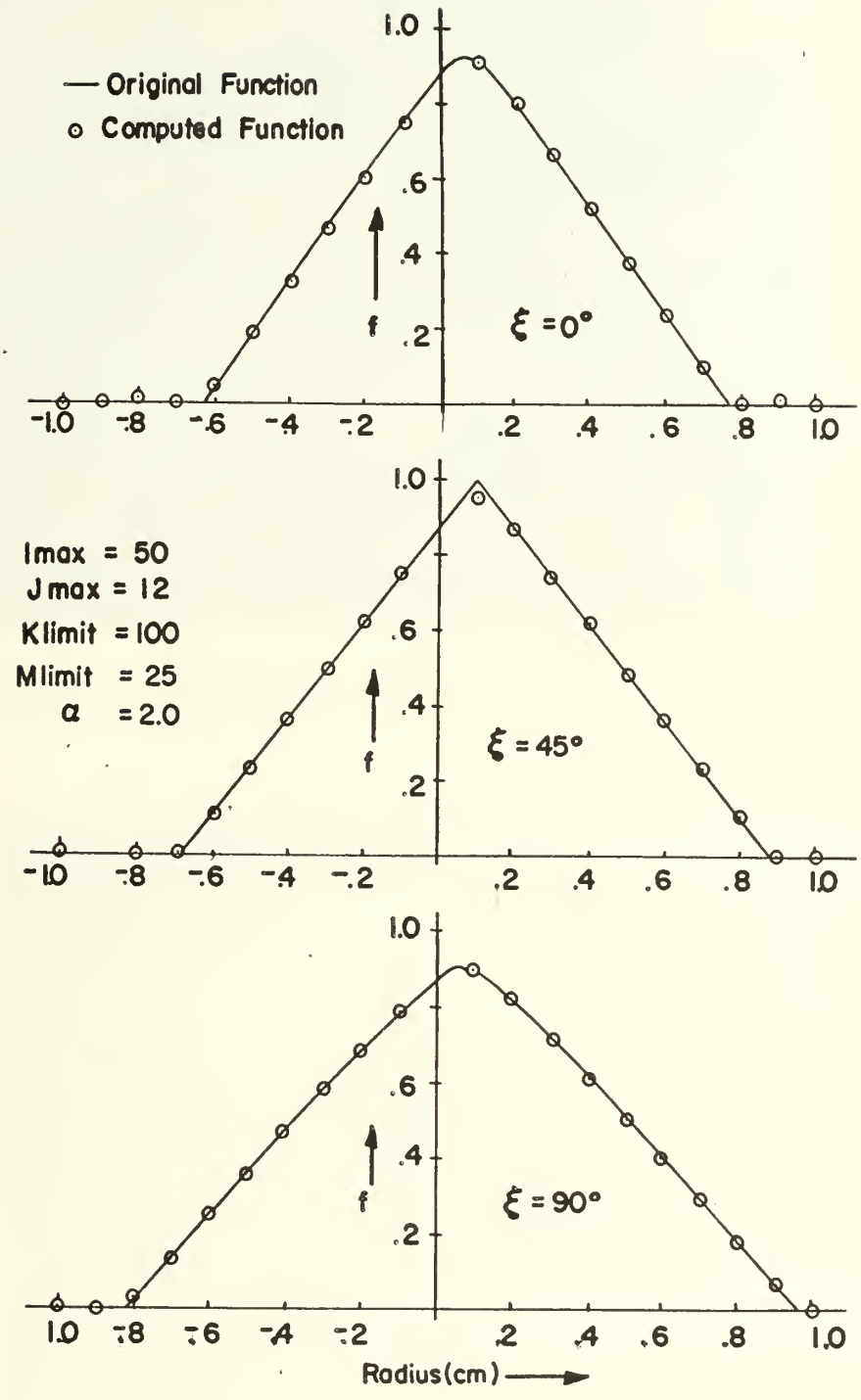

FIGURE 16 ASYMMETRIC TEST CASE—ELLIPTICAL CONE.

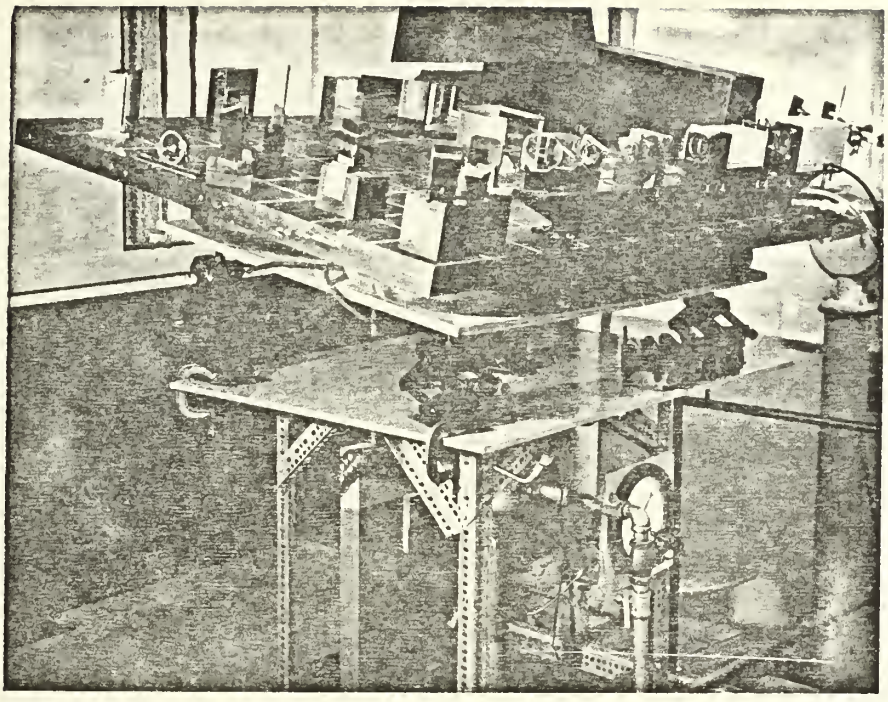

FIGURE 17. PHOTOGRAPH OF THE HOLOGRAPHIC TABLE

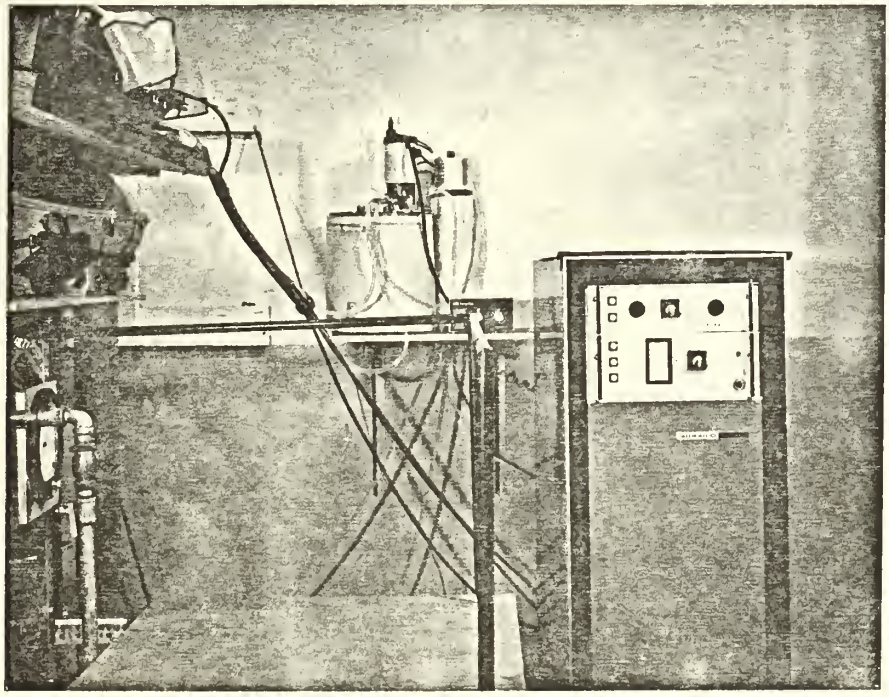

FIGURE 18. PHOTOGRAPH OF THE LASER POWER SUPPLY

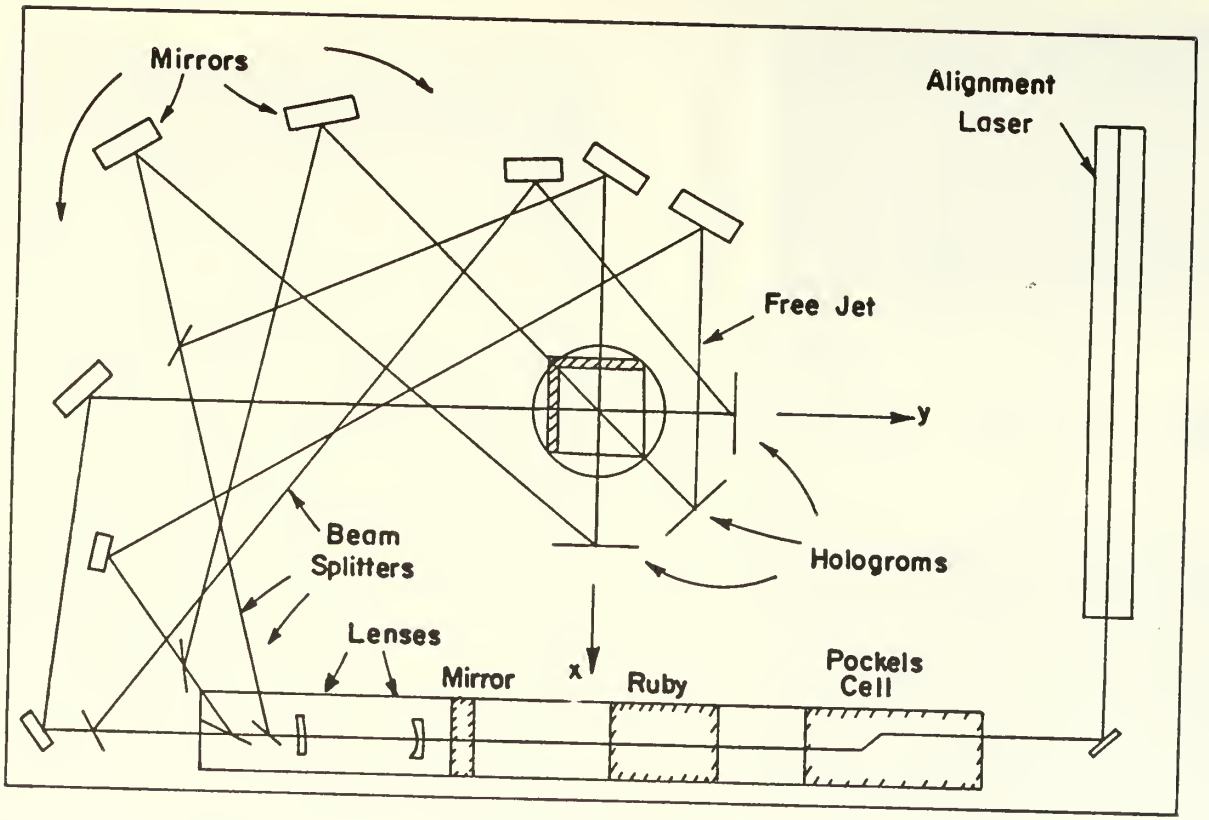

FIGURE 19. SCHEMATIC ARRANGEMENT OF HOLOGRAPHIC TABLE

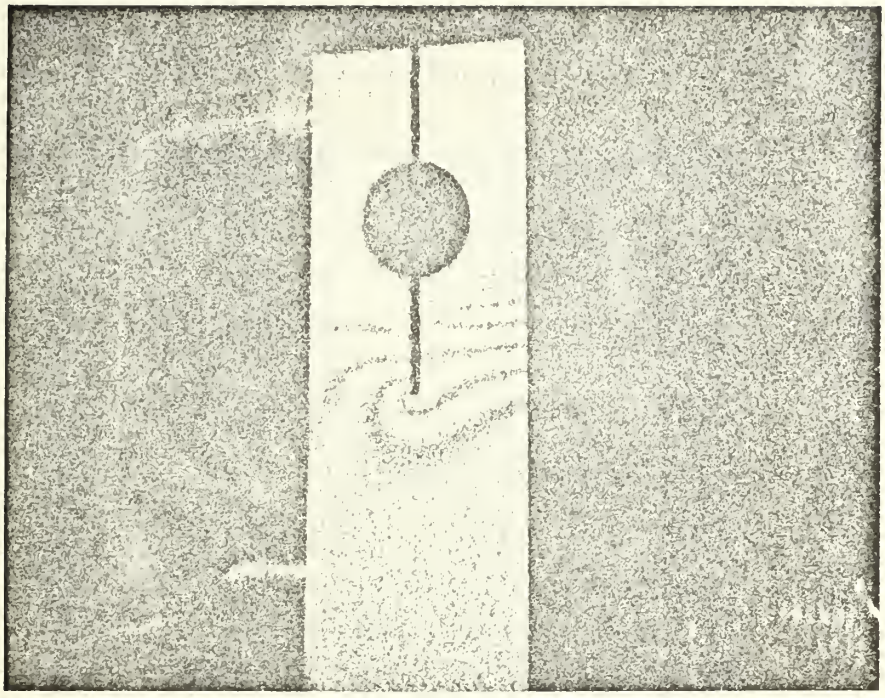

FIGURE 20. SURFACE INTERFEROGRAM OF A CLAMP UNDER STRAIN.

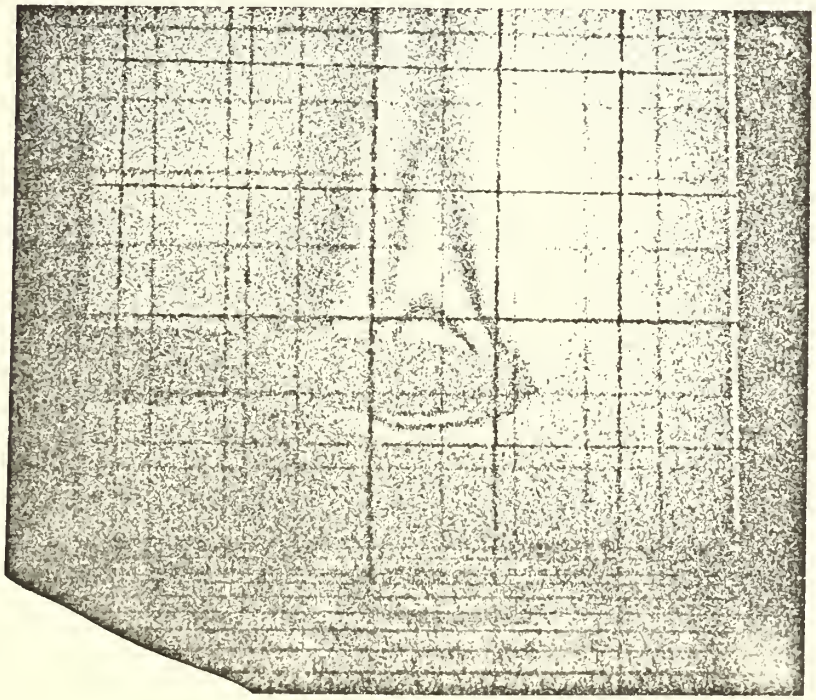

FIGURE 21. INTERFEROGRAM OF A CIGARETTE.

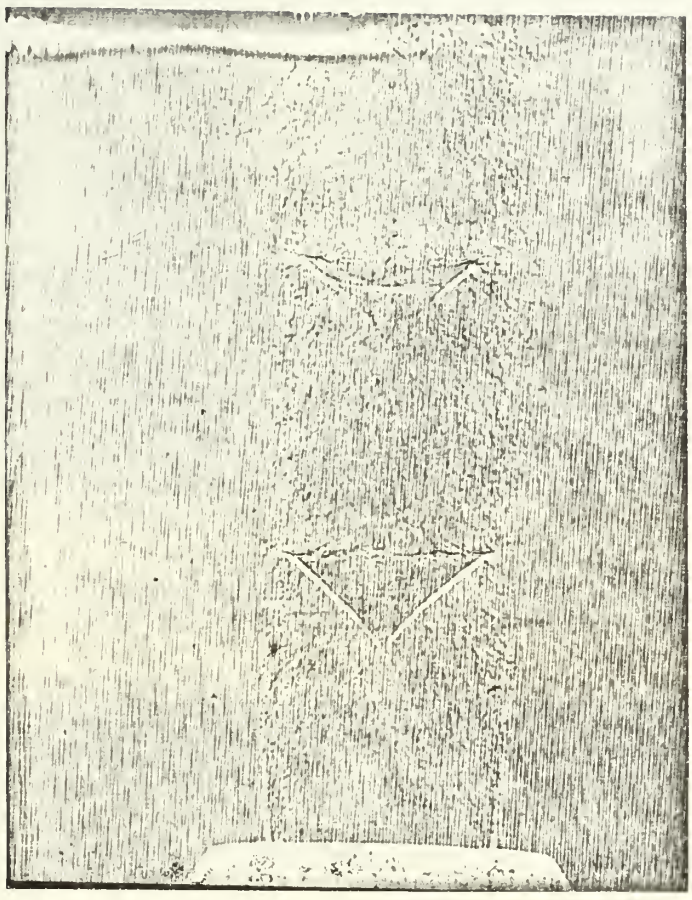

FIGURE 22. DIRECT PRINT OF A HOLOGRAM (DARK FIELD) SHOWING SHADOWGRAPH OF A FREE JET AT 35 PSIG.

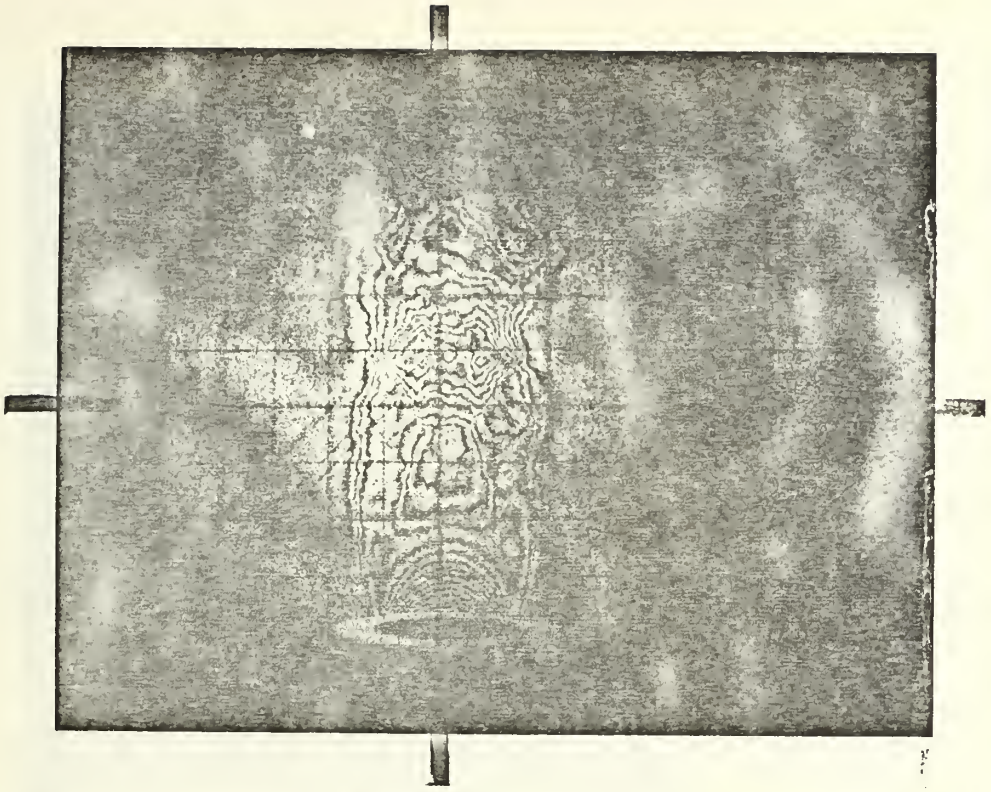

FIGURE 23. AXISYMMETRIC FREE JET, 60 PSIG. Z=3.0 CM

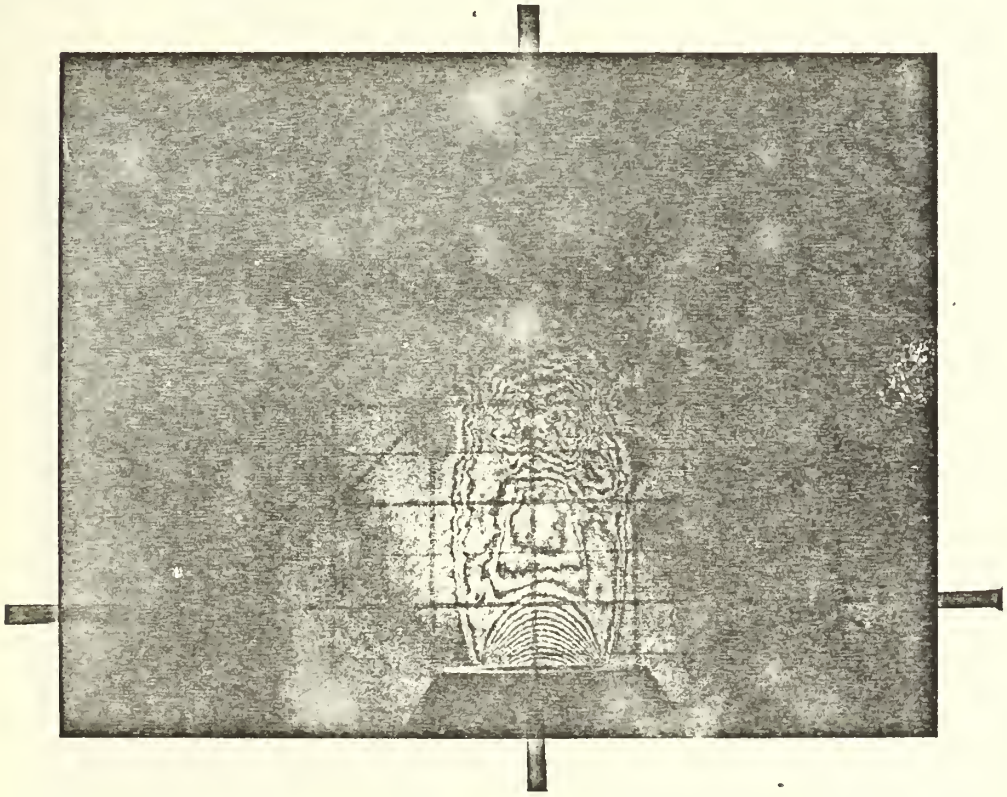

FIGURE 24. AXISYMMETRIC FREE JET, 60 PSIG. Z=1.0 CM.

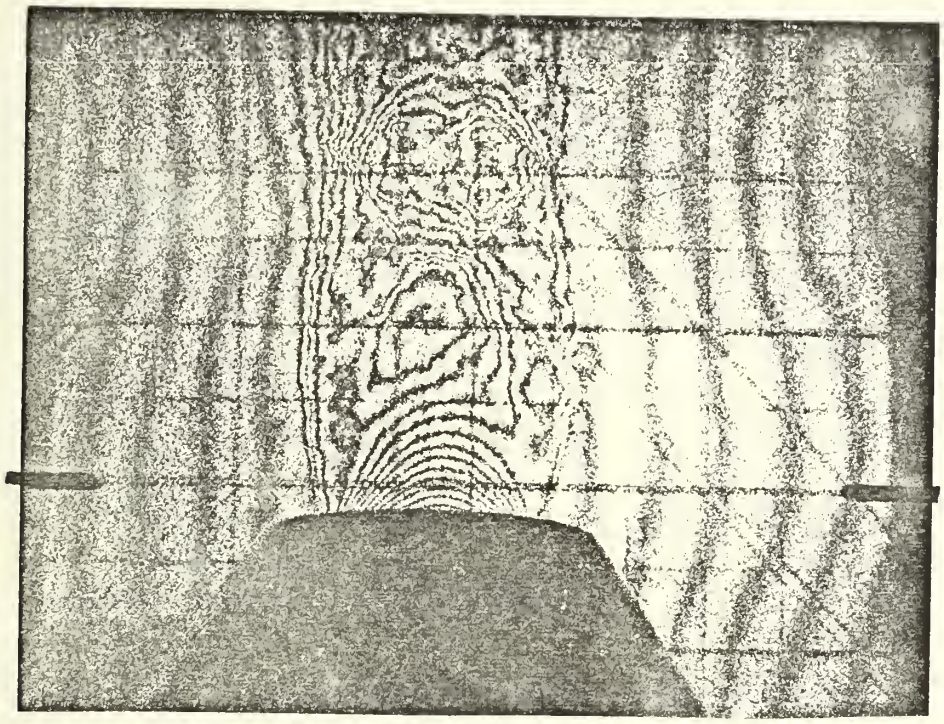

FIGURE 25. ASYMMETRIC SECTION OF A FREE JET, 60 PSIG., Z = .5 CM., 11° TILT, ξ = 5°

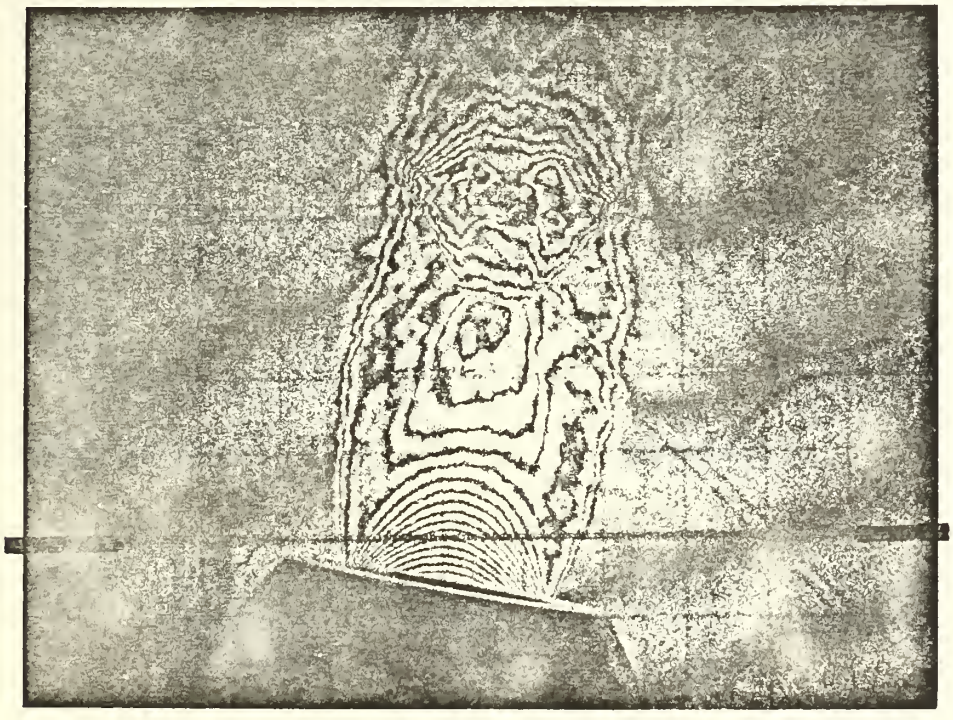

FIGURE 26. ASYMMETRIC SECTION OF A FREE JET, 60 PSIG., Z = .5 CM., 11° TILT, ξ = 85°

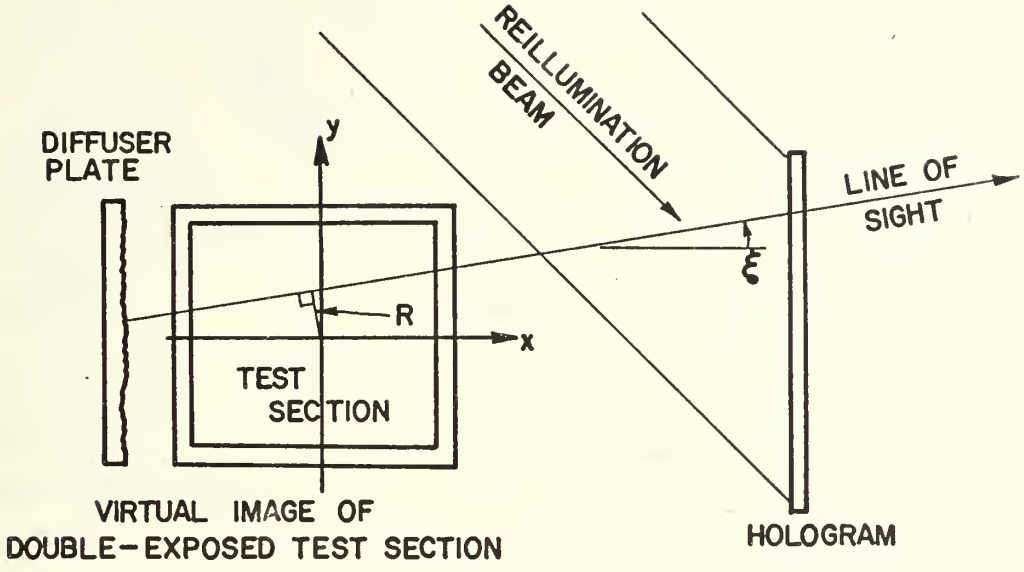

FIGURE 27: (R,ξ) COORDINATES FOR A LINE OF SIGHT THROUGH THE RECONSTRUCTED TEST SECTION

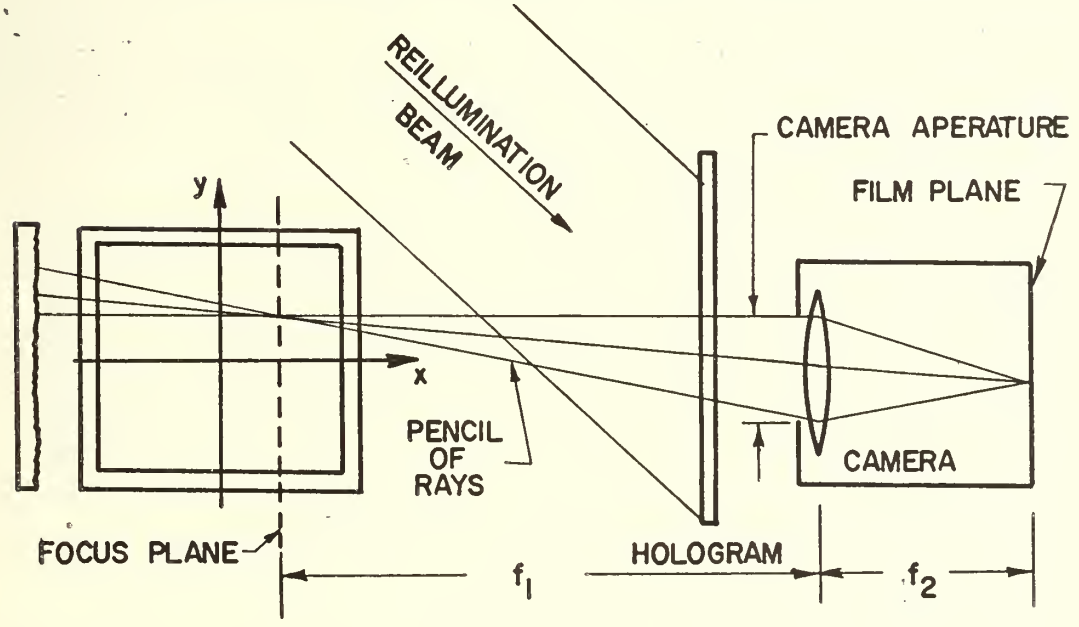

FIGURE 28: EFFECT OF APPERATURE SIZE AND FOCUS PLANE POSITION ON PENCIL SIZE OF RAYS ABOUT A LINE-OF-SIGHT RECORDED BY CAMERA.

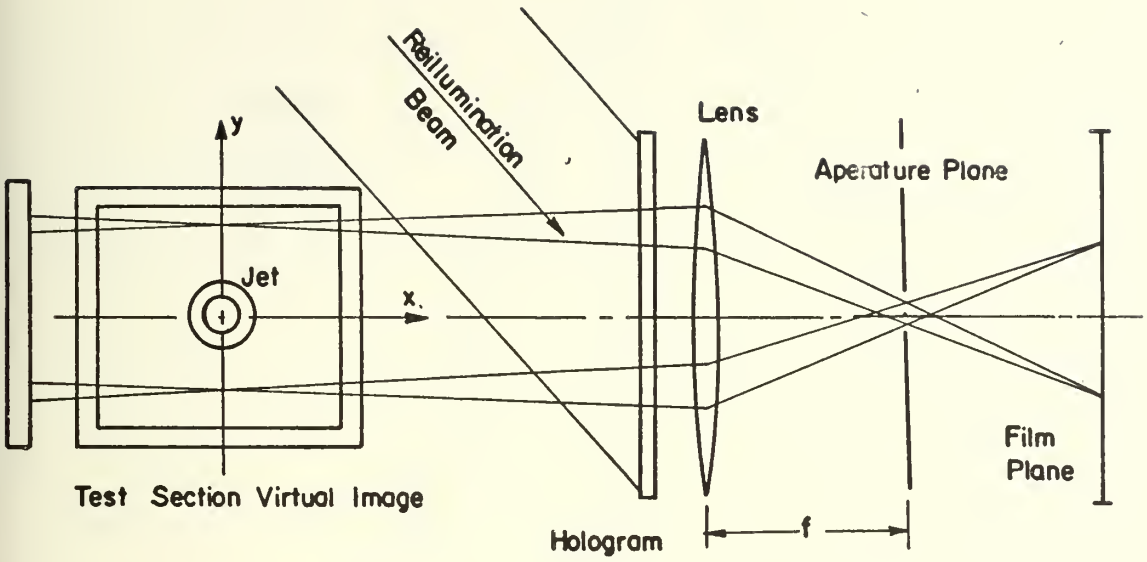

FIGURE 29 SPATIAL FILTERING TECHNIQUE FOR SELECTING PHOTOGRAPH OF CONSTANT ANGLE LINES OF SIGHT

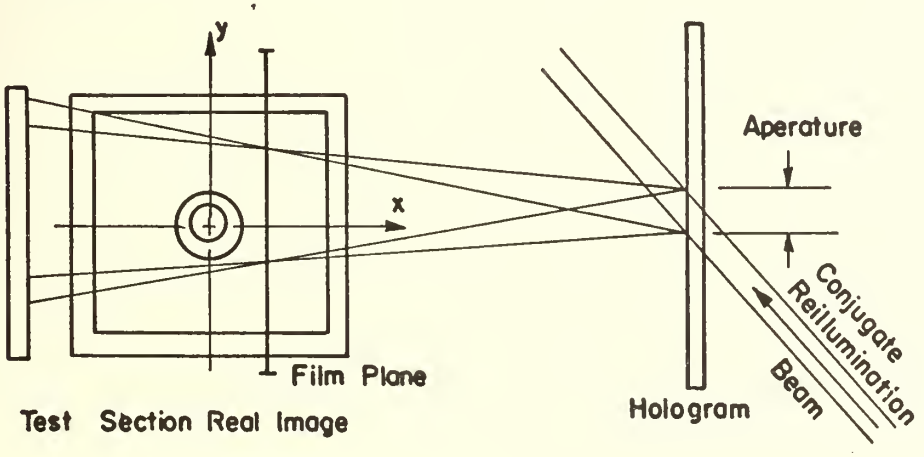

FIGURE 29a LENSELESS PHOTOGRAPHIC TECHNIQUE USING CONJUGATE REILLUMINATION BEAM OF SMALL DIAMETER

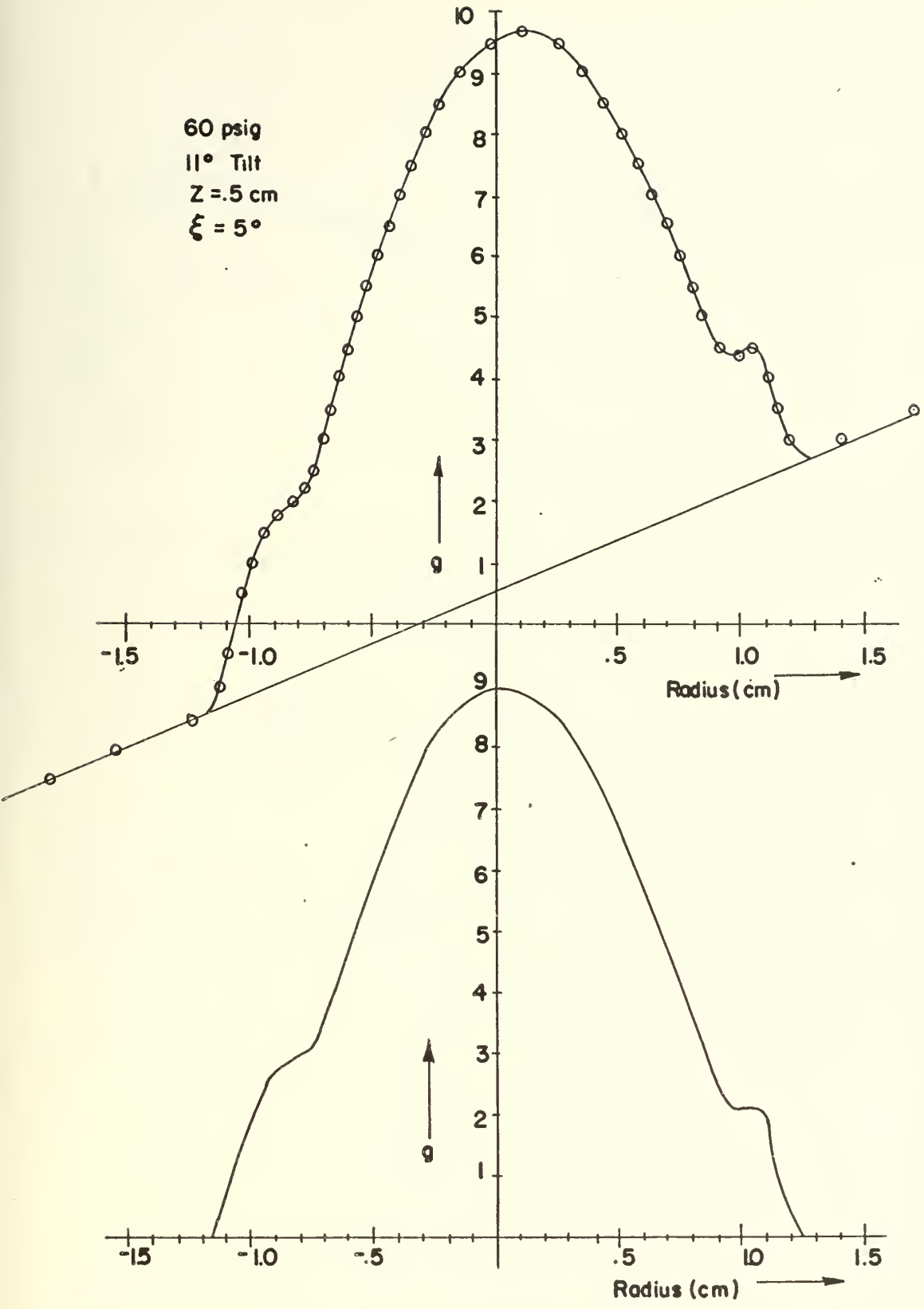

FIGURE 30 EXAMPLE GRAPHICAL WORK SHEET CURVES

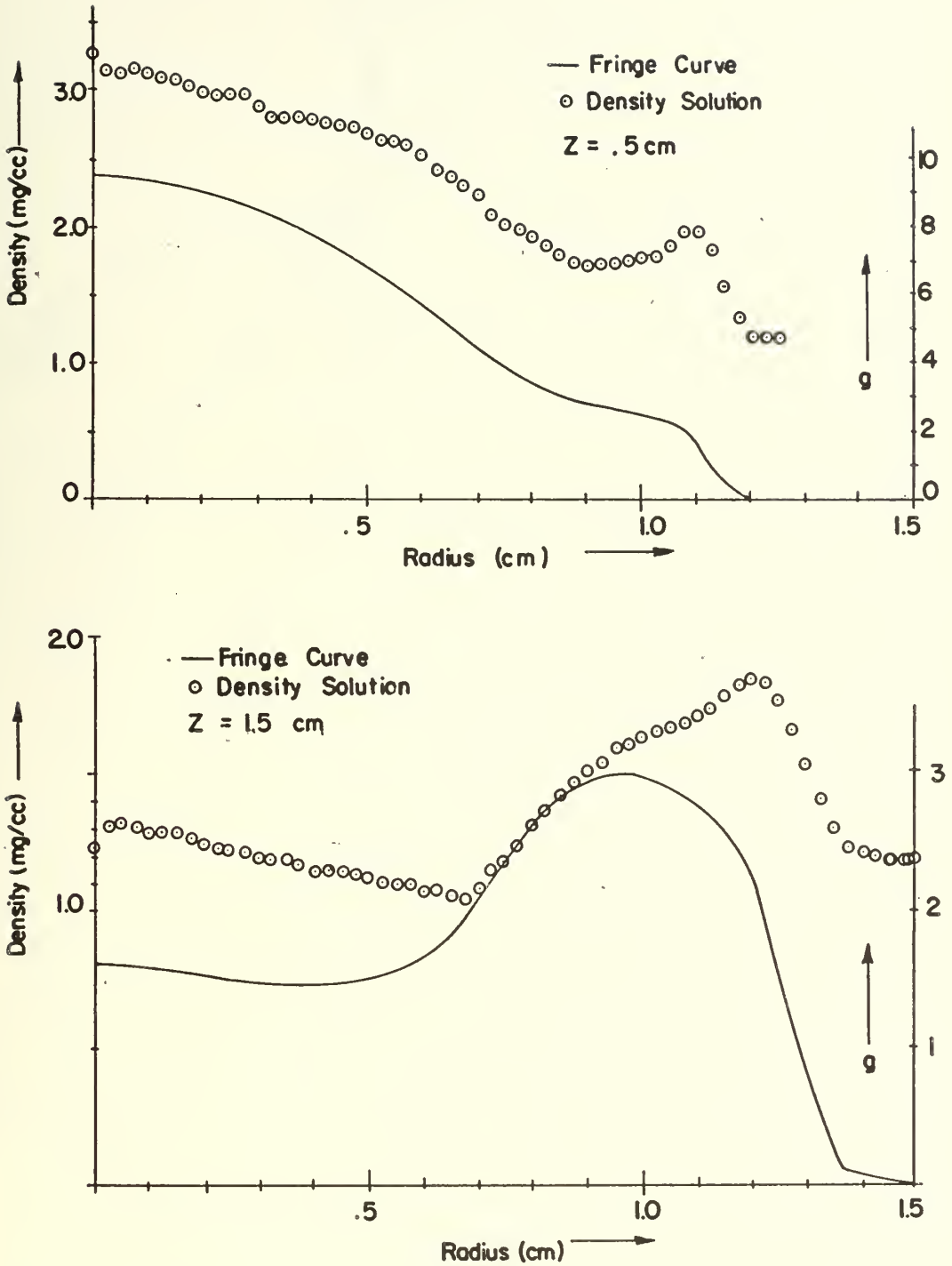

FIGURE 31 FRINGE CURVES AND CORRESPONDING SOLUTIONS FOR TWO STATIONS, AXISYMIC FREE JET, 60 PSIG

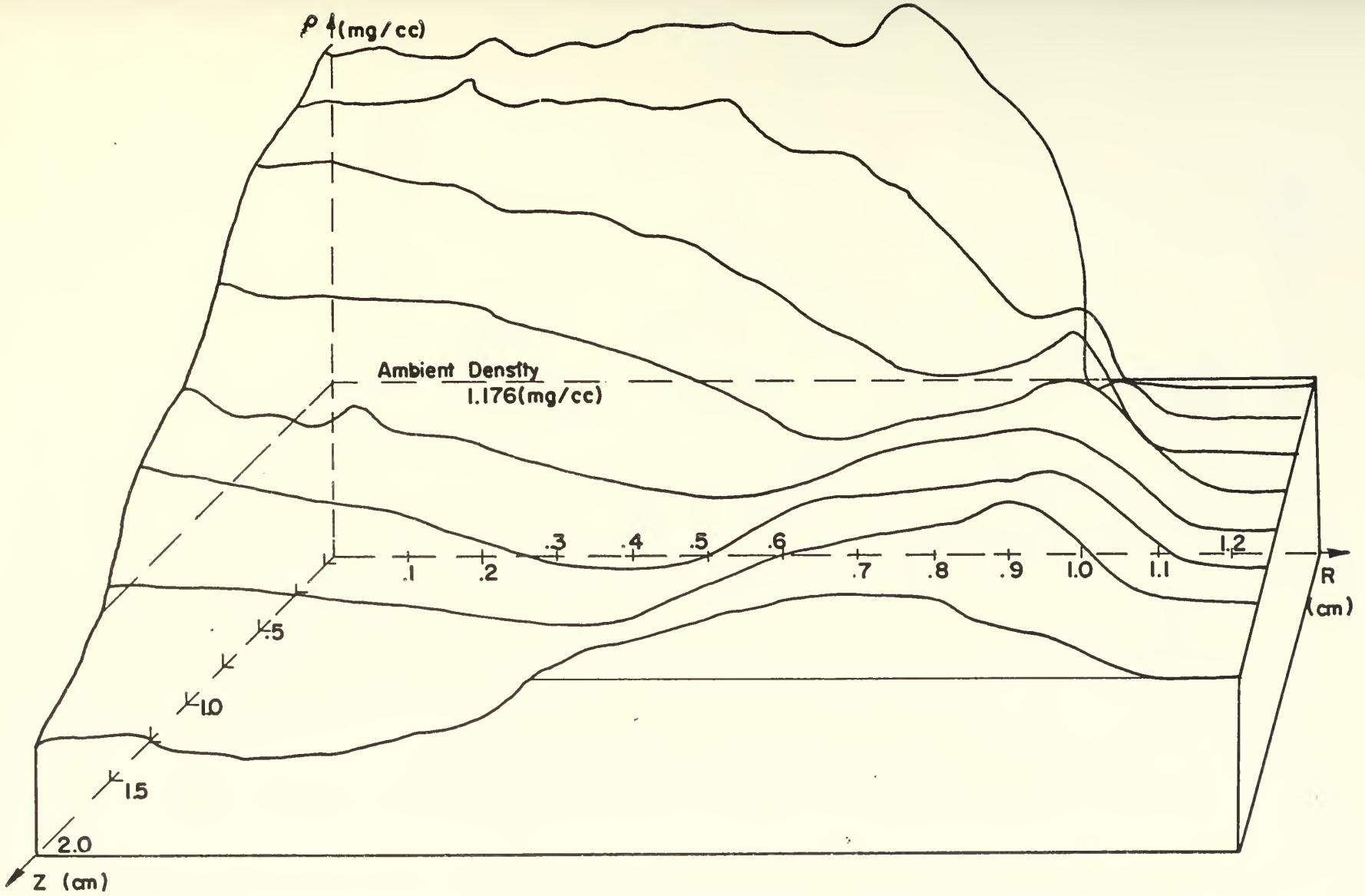

FIGURE 32 TOPOGRAPHICAL PLOT OF THE AXISYMMETRIC DENSITY SOLUTION OF A FREE JET AT 60 PSIG.

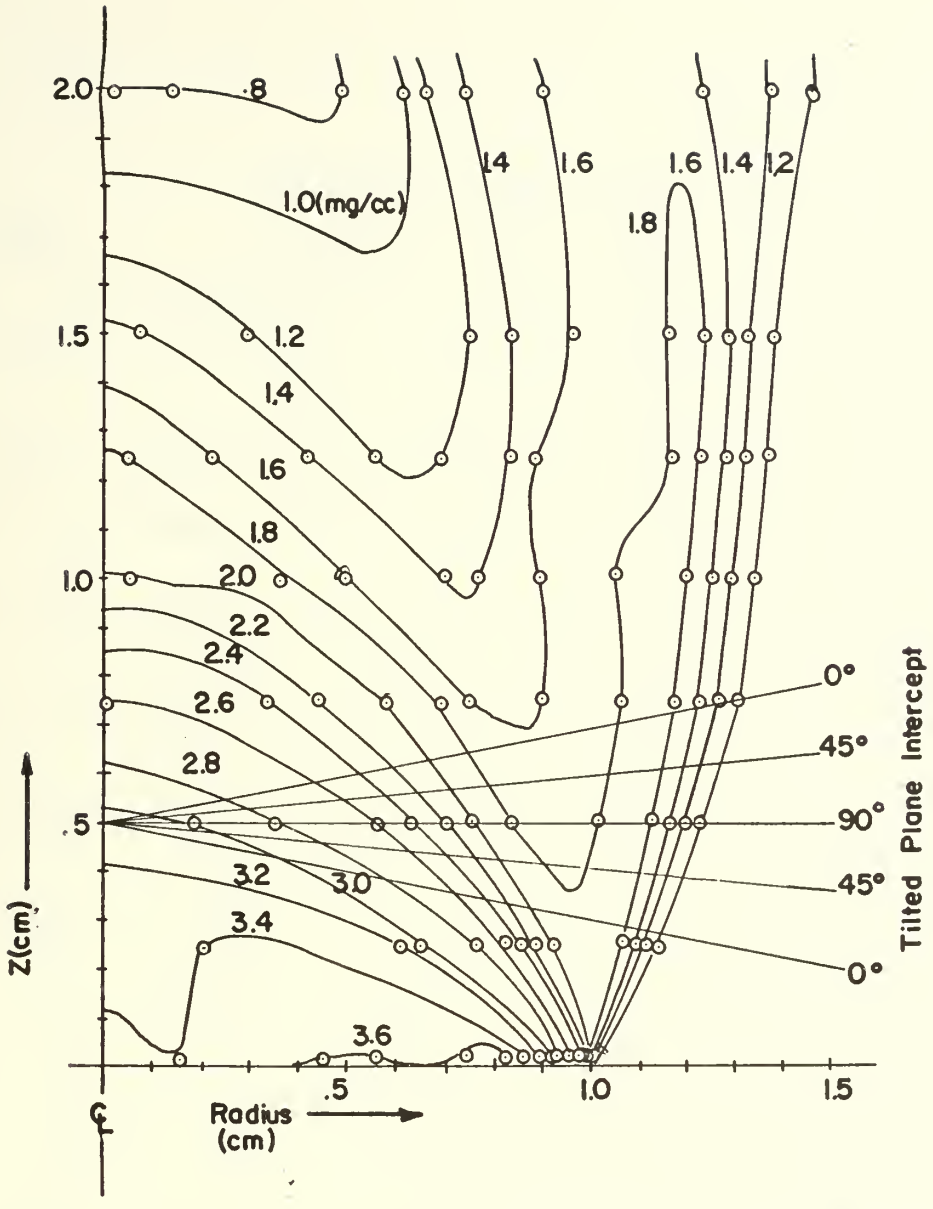

FIGURE 33. ISODENSITY LINE PLOT OF THE AXISYMMETRIC SOLUTION, 60 PSIG

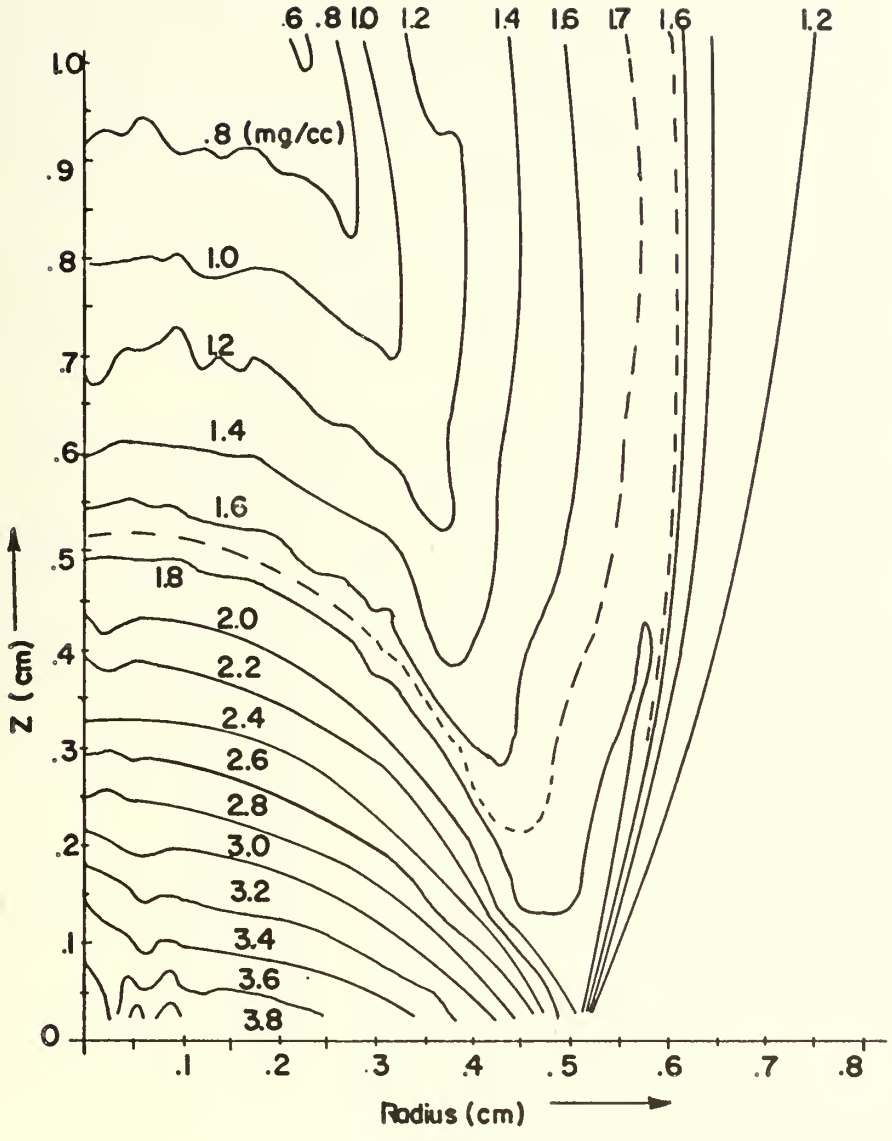

FIGURE 34 SKETCH OF THE AXISYMMETRIC DENSITY SOLUTION OF A 1.0 CM DIAMETER FREE JET AT 60 PSIG, FROM WINCKLER.

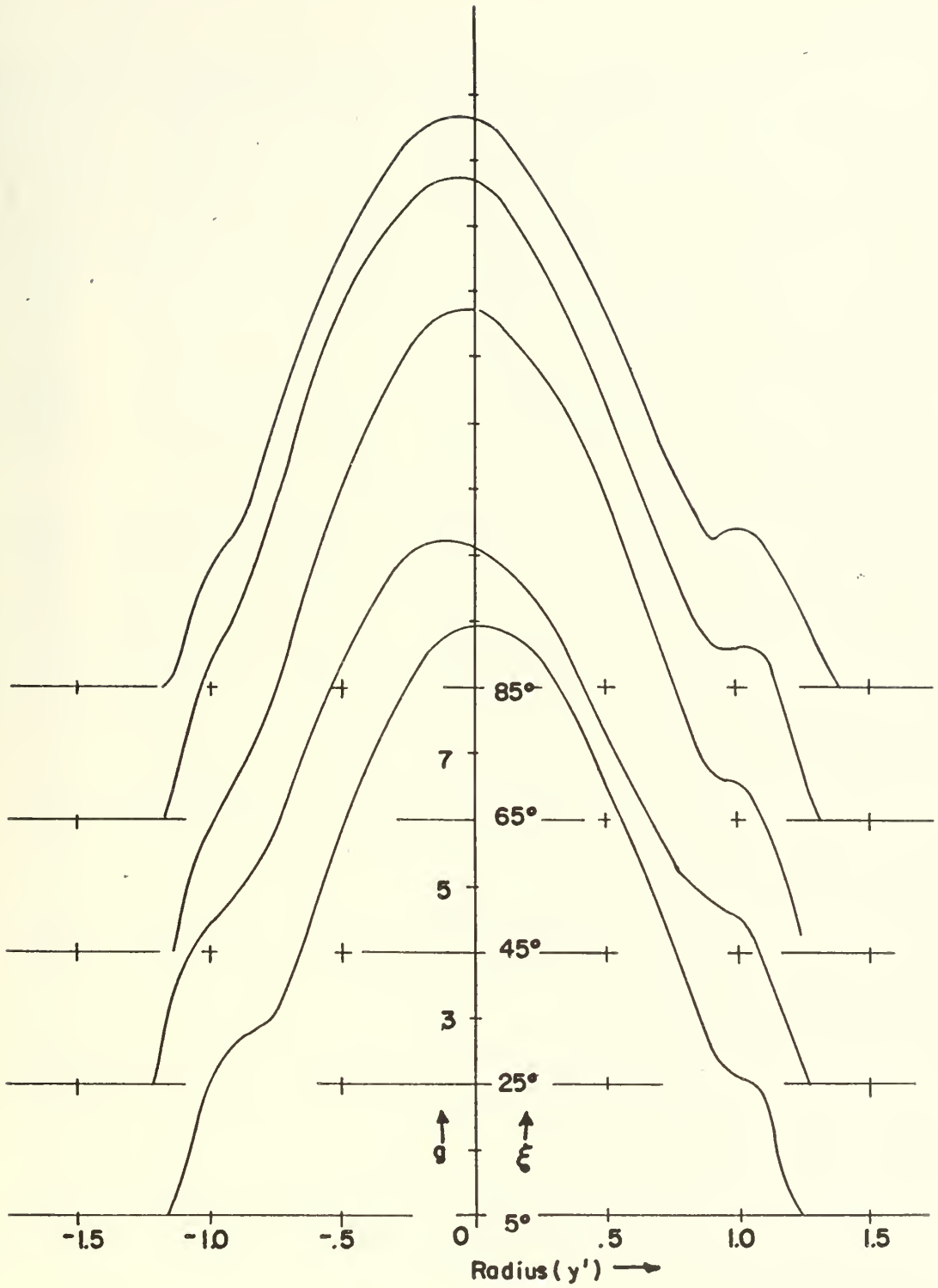

FIGURE 35. FRINGE NUMBER G CURVES OBTAINED FOR THE 11° TILT FREE JET AT 60 PSIG.

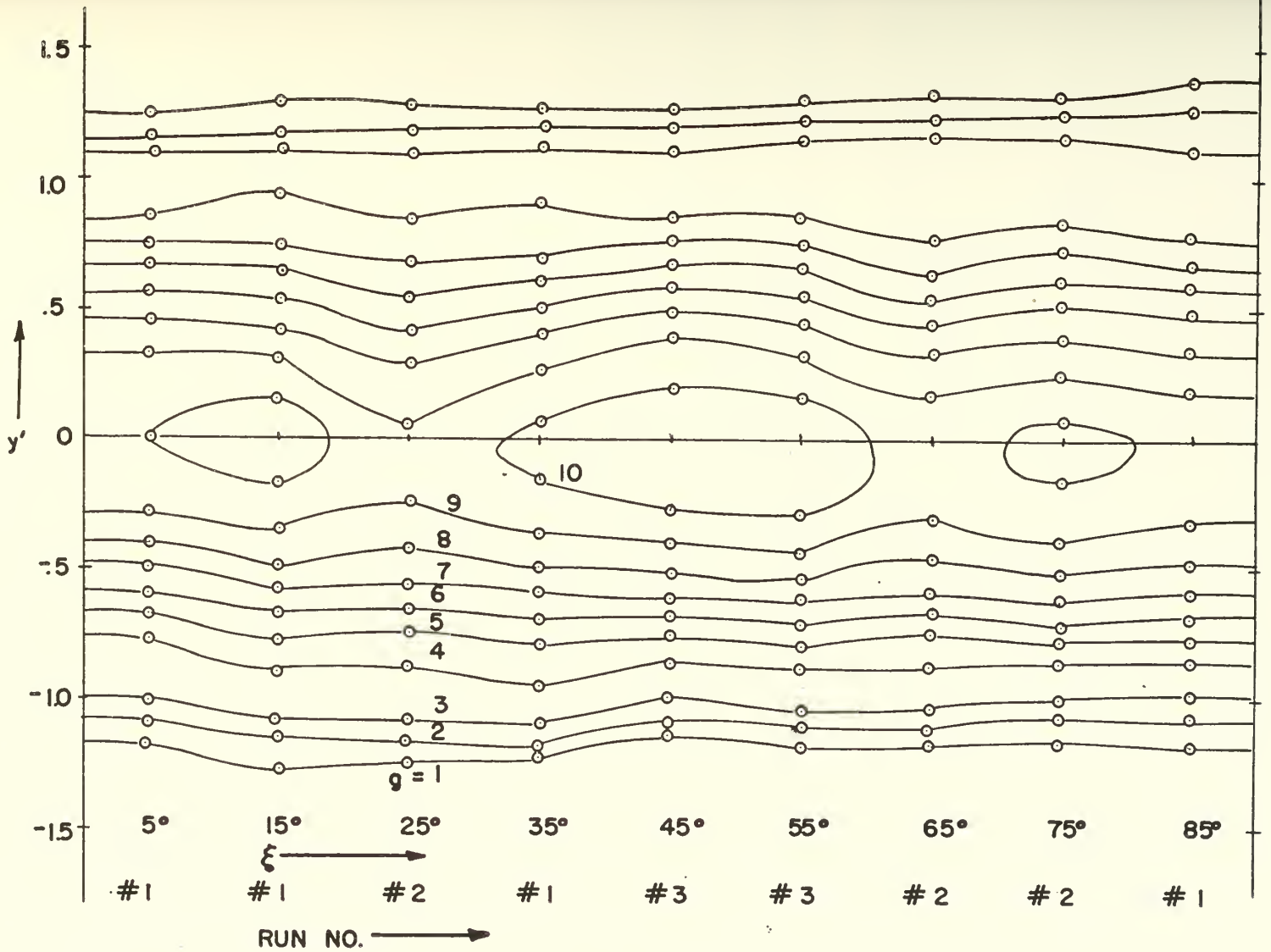

FIGURE 36. CONTOUR MAP OF THE FUNCTION G, 60 PSIG, 11° TILT

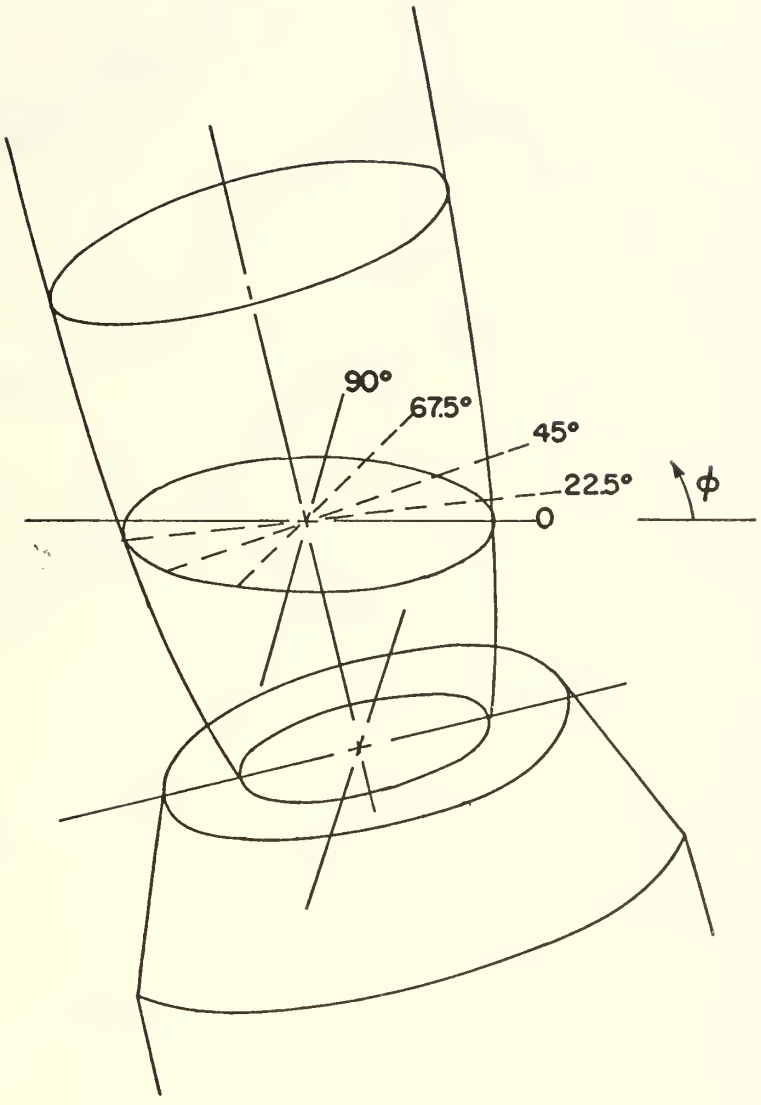

FIGURE 37 SKETCH OF TILTED PLANE SHOWING SOLUTION LINES.

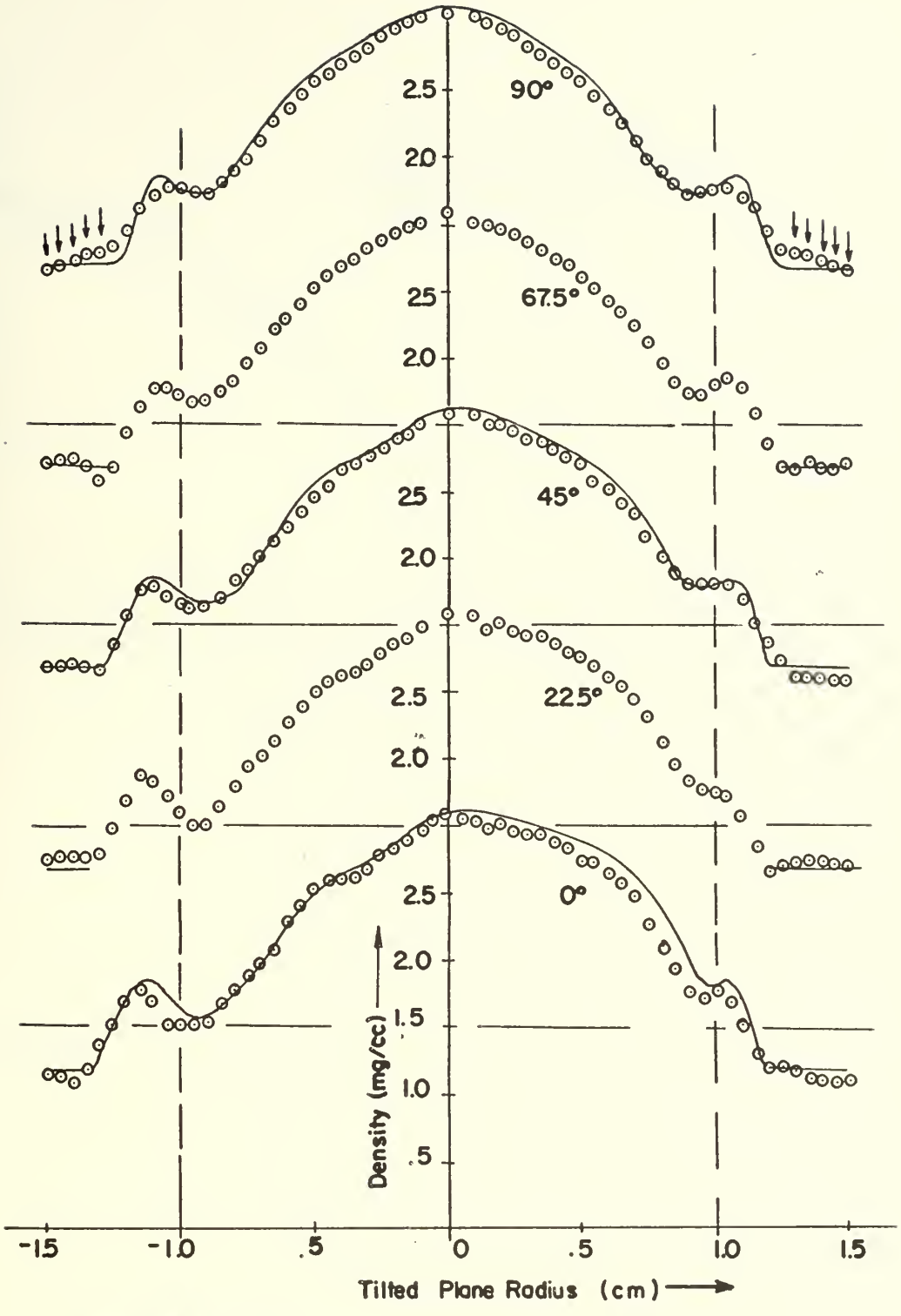

FIGURE 38 SOLUTION OF THE TILTED PLANE DENSITY ON FIVE DIAMETER LINES, 60 PSIG

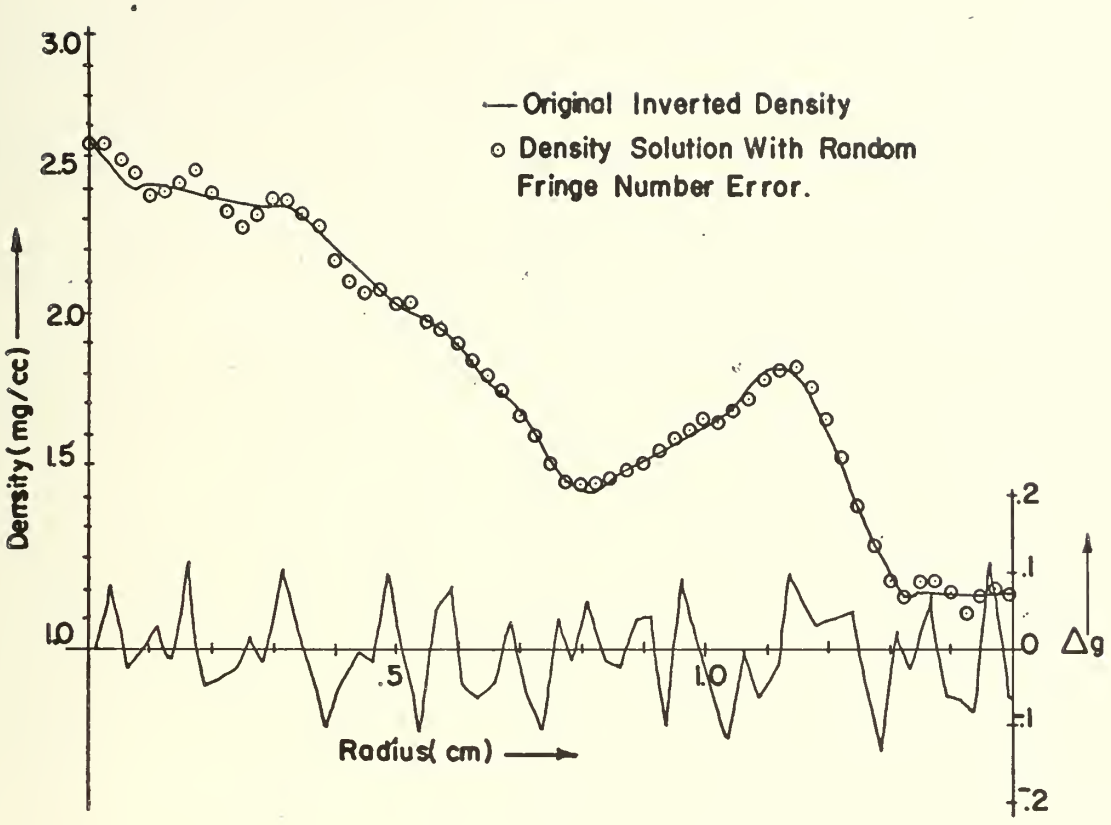

FIGURE 39 THE EFFECT OF RANDOM ERROR $\Delta_r G$ ON THE AXISYMMETRIC SOLUTION AT Z = .75 CM

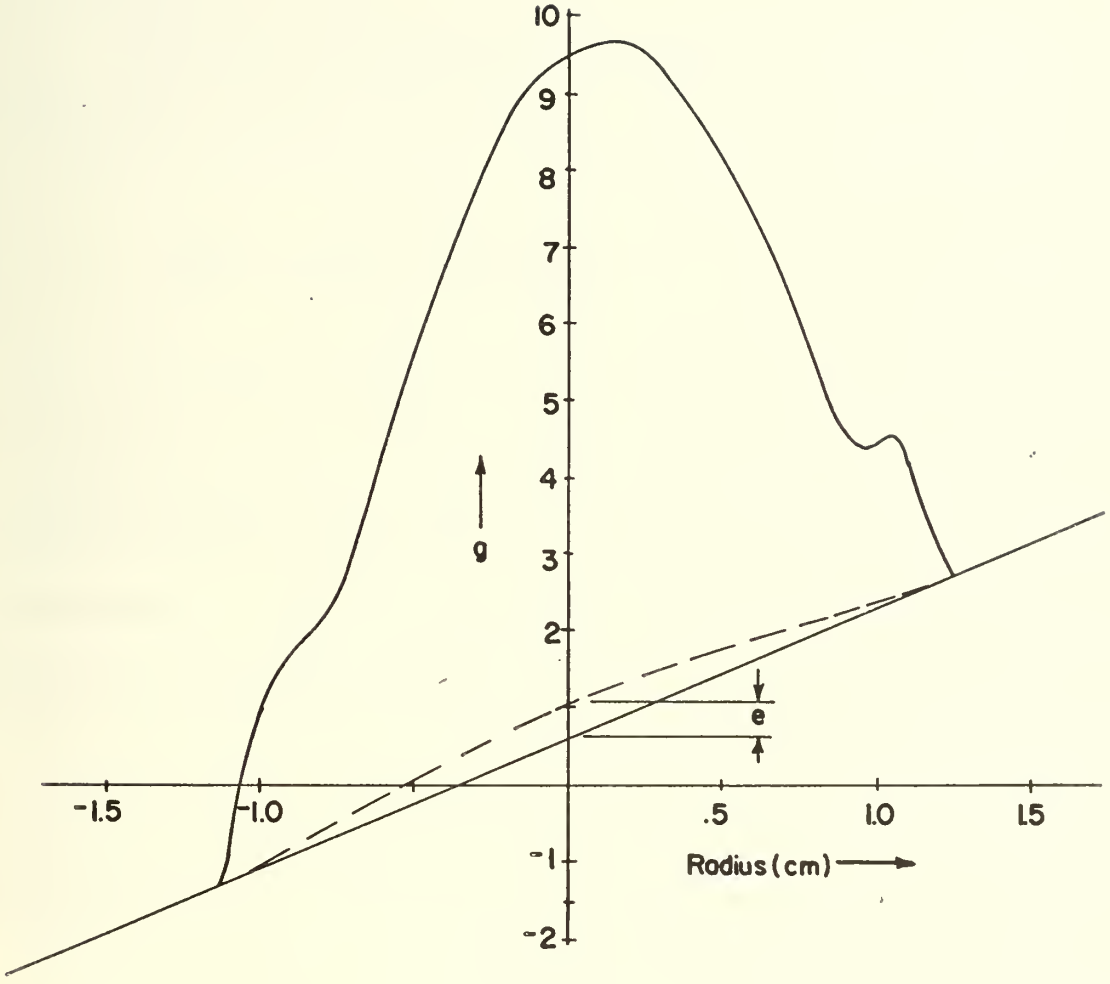

FIGURE 40 ILLUSTRATING POSSIBLE ERROR DUE TO NON-LINEAR BACKGROUND FRINGE.

APPENDIX A

OTHER INVERSION METHODS

1. The Abel Inversion

For the case of axisymmetry, the function f becomes invariant to a rotation of the coordinate system, and equation eleven reduces to:

$$g(y) = 2Q \int_{y}^{s/2} \frac{r f(r) dr}{(r^2 - y^2)^{1/2}} \qquad \text{(A1)}$$

where $r=(x^2 + y^2)$ 1/2, which is a form of Abel's integral equation with a well known solution. The Abel inversion has long been the standard method for reducing the data from axisymmetric interferograms. [Ladenberg, ed. 1954]. It has been well studied and optimized. Unmodified, the Abel solution overshoots the correct values when crossing shock wave discontinuities. Iterative methods have been developed which reduce this problem [Sangster and Shaw 1968]. In addition, there is the "reduced function" method previously mentioned.

2. The Generalized Abel Inversion

Recently a generalization of the Abel inversion by A. Pal has been reported [Cavanaugh, et.al. 1969] which applies to the two-dimensional function of equation eleven. It is in the process of being applied and evaluated at another laboratory. The method appears to have very similar application as the method utilized in this investigation.

3. Discrete Cubes

By representing the two-dimensional domain as a region of NXN, discrete cubes of constant values (see figure five), one can represent equation eleven as a set of N^2 linear equations in N^2 unknown cube densities. The complexity of the resulting matrix inversion problem increases with the increasing size of the N^4 matrix of coefficients, however, and the method is not presently competitive.

4. Rowley's Method

An analytical method involving a two-dimensional fourier transform has been reported recently [Rowley, 1969], but no application of the method is known to have been attempted.

APPENDIX B MATHEMATICAL FOUNDATIONS

The selection of a set of functions which are orthogonal and defined over the domain has been made on the assumption that there exists a polynomial transform relationship which corresponds to the relationship between the functions f and g. Consider the operator notation of equation (11):

$$g = Tf \tag{B1}$$

which has an inverse operation:

$$f = T^{-1} g \tag{B2}$$

One seeks a pair of orthogonal polynomial sets which, with an appropriate weighting function w, will satisfy the same transform operations.

Herlitz [1963], following a procedure from the diffraction theory of aberrations [Born and Wolf, 1959], for the cylindrically symmetric form of equation (11), and for the domain $r \leq 1$, used the Chebyshev functions [Madelung, 1953]:

$$V_k(y) = \sin^{-1}(k \cos^{-1} y) \tag{B3}$$

which satisfy the orthogonality relation

$$\int_{-1}^{+1} V_k(y)\, V_\ell(y)\,(1-y^2)\,dy = \int_{-\pi}^{\pi} \sin(k\theta)\, \sin(\ell\theta)\, d\theta = \frac{\pi}{2}\,\delta_{k\ell} \tag{B4}$$

and the Zernike polynomials, which have the form of the Legendre polynomials of argument $t = 2r^2 - 1$, and are orthogonal over the unit circle:

$$2(2k+1) \int_0^1 P_k(t)\, P_\ell(t)\, r\, dr = \delta_{k\ell} \tag{B5}$$

Using the Mehler integral [Whittaker and Watson, 1927]:

$$P_k(\cos\phi) = \frac{2^{\frac{1}{2}}}{\pi}\int_0^{\phi} \frac{\cos(k+\frac{1}{2})\psi\, d\psi}{(\cos\psi - \cos\phi)} \qquad \text{(B6)}$$

where $\cos\frac{\phi}{2} = r$ and $\cos\frac{\psi}{2} = y$, Herlitz reports the transform pair:

$$T P_k(t) = \frac{2}{2k+1} V_{2k+1}(y) \qquad \text{(B7)}$$

and

$$P_k(t) = \frac{2}{2k+1} T^{-1} V_{2k+1}(y) \qquad \text{(B8)}$$

Thus, by expanding f(r) in the Zernike polynomials with unknown coefficients

$$f(r) = \sum_{k=0}^{\infty} A_k P_k(t) \qquad \text{(B9)}$$

and applying the transform relation (B1), one gets:

$$g(y) = \sum_{k=0}^{\infty} \frac{2A_k}{2k+1} V_{2k+1}(y) \qquad \text{(B10)}$$

The orthogonality relationship (B4) allows the determination of the unknown coefficients

$$A_k = \frac{2k+1}{\pi}\int_{-1}^{1} g(y)\, V_{2k+1}(y)\,(1-y^2)^{\frac{1}{2}}\, dy \qquad \text{(B11)}$$

Application of the evaluated coefficients A_k to the expansion of equation (B9) yields the desired solution for the density.

Maldonado [1965], in a similar analysis for the axisymmetric case over the entire domain $r \leq \infty$, using a generating procedure attributed to Bhatia and Wolf [1954], obtained the orthogonal polynomials:

$$U_{2k}(\alpha x, \alpha y) = (-1)^k \frac{\alpha}{\pi^{1/2}} L_k(\alpha^2 x^2 + \alpha^2 y^2) \qquad \text{(B12)}$$

in which L_k are the Laguerre polynomials, defined over the interval $r \leq \infty$,

which have a transform relationship with gaussian weighting function:

$$\int_{-\infty}^{\infty} U_{2k}(\alpha x,\alpha y)\, e^{-\alpha^2 x^2}\, dx = \frac{1}{k!\, 2^{2k}}\, H_{2k}(\alpha y) \tag{B13}$$

where $H_{2k}(\alpha y')$ are the Hermite polynomials.

Hermite polynomials have the following orthogonality relationship:

$$\int_{-\infty}^{\infty} H_{2k}(\alpha y)\, H_{2\ell}(\alpha y)\, e^{-\alpha^2 y}\, dy = \frac{\delta_{k\ell}}{\alpha}\left[\pi 2^{2k} 2^{2\ell} (2k)!\,(2\ell)!\right]^{\frac{1}{2}} \tag{B14}$$

Expanding the function f in terms of the polynomials U:

$$f(x,y) = \sum_{k=0}^{\infty} C_{2k}(\alpha)\, U_{2k}(\alpha x,\alpha y)\, e^{-(\alpha^2 x^2+\alpha^2 y^2)} \tag{B15}$$

and applying the transform $g = Tf$, one obtains:

$$g(y) = \sum_{k=0}^{\infty} \frac{1}{k!\, 2^{2k}}\, C_{2k}(\alpha)\, H_{2k}(\alpha y)\, e^{-\alpha^2 y^2} \tag{B16}$$

Again, the application of the orthogonality relationship (B14) provides for the evaluation of the unknown expansion coefficients:

$$C_{2k}(\alpha) = \frac{\alpha\, k!}{(2k)!\, \pi^{\frac{1}{2}}} \int_{-\infty}^{\infty} g(y)\, H_{2k}(\alpha y)\, dy \tag{B17}$$

As one can see, the logical structure of the development is the same as the Herlitz expansion, with the exception that the domain is here defined over the entire x, y plane. The evaluation of the unknown density function is made by evaluating equation (B15) with the coefficients obtained from (B17).

The same procedure is used for the asymmetric case. The polynomials chosen are [Maldonado, 1966]:

$$U_{m+2k}^{\pm m}(\alpha x,\alpha y) = (-1)^k \frac{\alpha}{\pi^{1/2}}\left[\frac{k!}{(m+k)!}\right]^{\frac{1}{2}} (\alpha^2 x^2+\alpha^2 y^2)^{\frac{m}{2}}\, e^{\pm i m\phi}\, L_k^m(\alpha^2 x^2+\alpha^2 y^2) \tag{B18}$$

which have been chosen to have the property of "invariance in form" to a rotation of the coordinate system [Maldonado, 1965].

The "invariance in form" is demonstrated: suppose one makes a rotation of the axis by angle $\S$. The polynomial $U^{\pm m}_{m+2k}$ becomes:

$$U^{\pm m}_{m+2k}(\alpha x',\alpha y') = (-1)^k \frac{\alpha}{\pi^{1/2}}\left[\frac{k!}{(m+k)!}\right]^{\frac{1}{2}} (\alpha x'^2+\alpha y'^2)^{\frac{m}{2}} e^{\pm im(\phi-\S)} L^m_k(\alpha x'^2+\alpha y'^2) \quad \text{(B19)}$$

or:

$$U^{\pm m}_{m+2k}(\alpha x',\alpha y') = e^{\mp im\S}\, U^{\pm m}_{m+2k}(\alpha x,\alpha y) \quad \text{(B20)}$$

Clearly, the form of the polynomial has remained unchanged.

The polynomials, $U^{\pm m}_{m+2k}(\alpha x,\alpha y)$ have a gaussian transform of:

$$I^{\pm m}_{m+2k}(\S,dy') = \int_{-\infty}^{\infty} U^{\pm m}_{m+2k}(\alpha x,\alpha y)\, e^{-\alpha^2 x'^2}\, dx'$$

$$= \int_{-\infty}^{\infty} U^{\pm m}_{m+2k}(\alpha x',\alpha y')\, e^{\mp im\S}\, e^{-\alpha^2 x'^2}\, dx'$$

$$= \frac{e^{\pm im\S}\, H_{m+2k}(\alpha y')}{\left[k!(m+k)!\right]^{1/2} 2^{m+2k}} \quad \text{(B21)}$$

where the orthogonality relationship becomes:

$$\int_{-\pi}^{\pi} e^{\pm im\S}\, e^{\mp in\S}\, d\S \int_{-\infty}^{\infty} H_{m+2k}(\alpha y')\, H_{n+2\ell}(\alpha y')\, e^{-\alpha^2 y'^2}\, dy' =$$

$$\frac{2\pi^{3/2}}{\alpha}\left[(m+2k)!(n+2\ell)!\, 2^{m+2k}; 2^{n+2\ell}\right]\, \delta_{mn}\, \delta_{(m+2\ell)(n+2\ell)} \quad \text{(B22)}$$

The property of "invariance in form" preserves the form of the relationships for both the transform operation (B21) and the orthogonality condition (B22) during arbitrary rotations of the axes, $\S$.

A similar expansion of the function f in terms of the polynomial $U^{\pm m}_{m+2k}$ is:

$$f(x,y)=\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\epsilon_m\, C^{\pm m}_{m+2k}(\alpha)\, U^{\pm m}_{m+2k}\,(\alpha x,\alpha y)\, e^{-(\alpha^2x^2+\alpha^2y^2)} \tag{B23}$$

followed by a similar application of the transform operator g = Tf yields:

$$g(y',\xi)=\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\epsilon_m\left[k!(m+k)!\,2^{2(m+2k)}\right]^{-\frac{1}{2}} C^{\pm m}_{m+2k}(\alpha)\, e^{\pm im\xi}\, H_{m+2k}(\alpha y')\, e^{-\alpha^2y'^2} \tag{B24}$$

from which an application of the orthogonality relationship yields an evaluation of the unknown coefficients $C^{\pm m}_{m+2k}$:

$$C^{\pm m}_{m+2k}=\frac{\alpha}{2\pi^{3/2}}\left[\frac{[k!(m+k)!]^{\frac{1}{2}}}{(m+2k)!}\right]\int_{-\pi}^{\pi}\int_{-\infty}^{\infty} g(y',\xi)\, H_{m+2k}(\alpha y')\, e^{\mp im\xi}\, dy'd\xi \tag{B25}$$

which allow the evaluation of the expanded function f of equation (B23).

The progressive application of the same logical sequence, which has been given, demonstrates the development of this method of inversion by orthogonal polynomial expansion.

APPENDIX C

HOLOFER,
A FORTRAN IV COMPUTER PROGRAM FOR INVERTING THE LINE INTEGRAL DATA OF HOLOGRAPHIC INTERFEROMETRY FOR ASYMMETRIC FIELDS

1. The Description of Input Parameters
2. A Sample Terminal Dialogue
3. The Terminal Executive Files
4. The Program
5. The Input Data Files Used in the Experiment

1. The Description of Input Parameters:

CMS — selects CP/CMS (terminal) or OS/MVT (batch) computer operation of the program. The selection is made by choice of the proper CMS definition card in the calling routine HOLOVERT.

IMAX — is the number of intervals of the input data in the radial direction, from y'= -S/2 to +S/2. 1 IMAX 201.

JMAX/2 — is the number of intervals of the input data in the azimuthal direction, from $\xi = 0^{o}$ to the edge of the orthogonal subinterval. $0 \leq JMAX/2 \leq 36$, if JMAX/2 is set to zero, the value of JMAX will default to 1. This applies to the axisymmetric case where only one line of data is needed.

NOTE: The dimension statements for G and GA in HOLOVERT set a limiting value of IMAX*JMAX at 5151. This would be exceeded if both values were individually set to their maximum. Only the dimension statements in HOLOVERT need be changed to change program dimensions.

KLIMIT — sets the number of B and D coefficients which will be computed corresponding to each k index of the double series.
$1 \leq KLIMIT \leq 1000$ for asymmetric (SYM = 0)
$1 \leq KLIMIT \leq 2000$ for symmetric (SYM 0)

MLIMIT — sets the number of KLIMIT sets of B and D coefficients which will be computed corresponding to each m index of the double series.
$1 \leq MLIMIT$. MLIMIT may be any size, limited only by the disk space available for storage. Values greater than 50 are not desirable, however.

KEXTRA — is the number of extra terms that will be included in the evaluation of the k series after the term size has failed the epsilon criterion.
$0 \leq KEXTRA \leq$ KLIMIT. This value insures that the term size has stabilized at a small value prior to truncating the evaluation. When the evaluation fails to converge prior to reaching KLIMIT terms, the last KEXTRA terms are averaged.

MEXTRA the number of extra M series that will be evaluated after the K series evaluations have become small. MEXTRA has the same effect upon the m term evaluation as does KEXTRA on the K series. $0 \leq$ MEXTRA $\leq$ MLIMIT.

ALPHA the scale factor for the Hermite and Laguerre polynomials. $1.0 \leq$ ALPHA ≤ 5.0
Maldonado, et.al. have shown that the factor Alpha has a significant effect upon the convergence criteria of the series. The guide line given is to match the edge slopes of the G function to a gaussian curve: $G = e^{-\alpha^2 y'^2}$
Then one uses the value α which matches. For the G functions used in this investigation, the variable SIZE has been adjusted to keep the function edges near .8 on the unit circle. ALPHA = 2.0 has been satisfactory for the experimental inversions.

SIZE defines the diameter of the region from which data is taken, i.e. the diameter of the inversion circle. SIZE has been 2.5 or 3.0 for all experimental inversions, 2.0 for all mode 1 test runs.

EPS is the convergence criterion. When the Nth term of the series becomes less than EPS times the partial sum to date, the KEXTRA or MEXTRA terms begin counting extra terms. If the (N+KEXTRA) term remains less than EPS*SUM, the series is truncated. EPS=.0001 was used in most inversions.

RHO-INF is the atmospheric density, in mg/cc., entered as an input for the density conversion of the output data. 1.176 mg/cc. was used for the experimental inversions.

LAMBDA is the wavelength of the hologram source light in angstroms, λ=6941.0 A for ruby laser holograms. λ =6328.0 A for helium-neon holograms.

BETA is the first term binomial expansion coefficient of the relation between index of refraction and density. BETA=.000291 for red light (6900A) BETA =.000293 for yellow-green light (5500A).

MODE selects the manner in which the G-array will be obtained. The three modes are described in the text and in the subroutine comment cards of the program. In addition to modes 1, 2, and 3, the parameter may be modified. Mode = -1, -2, or -3 will utilize subroutine FIELD2 which generates the B and D coefficients individually as needed. It requires much more time, but does not use offline disk space. MODE = 11, 12, or 13 will assume the B and D coefficients are already stored on the disk, will bypass subroutine BDGEN and use the previously computed coefficients. This provides a restart capability to the program. Once the coefficients have been generated, one can vary the output parameters for repeat inversions.

SYM selects the symmetry paramenters of the field to be inverted. SYM is called JSYM in the program.

SYM = 0 - completely asymmetric field

SYM = 1,2,3,4.... - 1,2,3,4... regularly spaced planes of symmetry exist in the field, aligned with plane number one on the X-Z plane.

SYM > JMAX - an axisymmetric field.

POINTS is the number of points to be inverted on each sampling line. POINTS ≥ 1

LINES is the number of lines of points through the field to be inverted LINES ≥ 1

DIAGNOS provides a series of levels of diagnostic printout. It is helpful if errors show up in the program to printout the parameters as they are computed. No explanation will be made here, as the use of this parameter involves detailed study of the program anyway. DIAGNOS = 0. for normal operation.

STD. DEV. provides the ability to add a random error of given standard deviation to the add-on function G-array. The resulting inversion will reflect the effect of random errors to the data. This parameter allows some degree of error estimate to the experimental data.

STD. DEV. = 0.0 for normal operation

STD. DEV. = .0625 for $\pm$ 1/8 fringe number error in the data.

PHIZERO is the angle in degrees, which the first line to be inverted will make with the X axis. PHIZERO =0.0 for most uses.

DELPHI is the angular incrementation, in degrees, that successive inversion lines will make with the X-axis. The second line will be at PHI=PHIZERO+DELPHI, etc.

YPZERO is the y' position of the first line to be inverted, in the primed coordinate system rotated by angle PHI

YPRANGE is the range over which y' will uniformly vary for the number of lines to be inverted. If one sets DELPHI = 0. and YPRANGE ≠0. A cartesian array of points will be selected, if DELPHI≠0. and YPRANGE=0. A cylindrical array of points will be selected.

XPZERO is the position of the beginning of each line to be inverted (in the primed coordinate system).

XPRANGE is the range over which the number of points selected will range, beginning at XPZERO.

TST.FUN is the number of the function to be inverted. In mode 1 this number corresponds to the function number in subroutine FUNCT. In modes 2 and 3 it serves as a run identifying number on the output.

ADD.FUN is the number of the function in FUNCT to be used as an add-on function. ADD.FUN=0. will default the use of an add-on function.

GARRAY is the output parameter for printout of the G-array used. GARRAY =

0. -- no printout.
1. -- prints the G-array.
2. -- prints the G-array with a pause for paper spacing on the terminal prior to printing.
3. -- prints both G-array and the add-on G-array.
4. -- prints both G-arrays with terminal pauses for spacing.
-1. -- punches the G-array on a deck of cards and prints the G-array
-3. -- punches the modified G-array sum of both on a deck of cards, then prints both G-arrays separately.

GRAPH prints a graph of each line of the G-array. GRAPH =

0. -- no graph.
1. -- a graph of G-array for each J.
2. -- same as 1., with terminal pause.
3. -- both G-array and add-on G-array are on each graph.
4. -- same as 3., with terminal pause.

LIN.PRT prints the inversion results LIN.PRT =

0. -- G-array is set up, inversion results are not printed. Inversion is not performed unless MAP.BND=0.

1. -- prints the inversion results a line at a time.
2. -- prints the inversion results with a terminal pause prior to each line of points, for paper spacing. It also sets a terminal pause prior to MAP output, if MAP is used.

MAP.BND prints a map of the inversion array on the printer. Uses library subroutine MTMPII. MAP.BND=

0. -- map is not printed.
$\neq$0. -- the number used defines the interval between contours of the function f on the map, e. g. 0.2.

A,B,C,D,E,P are variable function parameters for the functions in subroutine FUNCT, as used in mode 1 inversion.

S,T,U,V,W,Q are variable function parameters for the functions of subroutine FUNCT, as used for the add-on function.

PHISYM is an input angle entered in the data cards for mode 2 or 3 operation that defines a rotation between the inversion coordinate system and the laboratory, or output coordinate system, e.g. if the plane of symmetry of the function was at 5° in the laboratory system, one would enter 5.0 in the data cards. Output would then be in the laboratory coordinate system. Raw data for mode 2 is entered in the laboratory coordinate system; G-array data for mode 3 is entered in the data plane coordinate system.

XO,YO are the laboratory coordinates of the estimated center of the function, entered in the data cards used in mode 2, to provide a well-centered inversion. Output is in laboratory coordinates.

INPUT DATA for batch operation, input data cards are read in order:

1) 7 cards of input data, format 89 of calling program HOLOVERT; data corresponds exactly to parameters in the sample dialogue.

2) IF USED: data cards for numerical function, formats 88, 89 of subroutine FREAD.

3) IF USED: data cards for numerical add-on function, formats 88, 89 of subroutine FREAD.

4) input data for mode 2 or mode 3

   -mode 2: formats 59, 58 of subroutine SHEET;
   if NCODE.GE.1: format 29 of subroutine SIM.
   -mode 3: formats 39, 38 of subroutine READ.

5) a blank card if another data set follows, otherwise a FINISH card; format 60, HOLOVERT.

   For terminal operation, input data 2), 3), and 4) are entered in the above formats in separate files and given optional filenames/filetypes. The organization of files for execution is handled by exec file HOLOFER.

   A sample terminal dialogue follows.

2. A Sample Terminal Dialogue on the Cambridge Monitor System.

```
holofer
ERASE FILE FT03F001 *
ERASE FILE FT04F001 *
SPLIT INPUT DATA FILE FT04F001 1 EOF
EOF REACHED
FILE MODIFIED

ANSWER YES OR NO...

DO YOU WISH INFO TYPEOUT?
no

DO YOU WISH INPUT DATA TYPED OUT OR CHANGED?
yes

THE INPUT VALUES FOLLOW:
--1) IF CHANGES ARE DESIRED TYPE NEW VALUE(S) UNDER OLD
--2) IF NO CHANGES, HIT SPACE BAR AND RETURN
--3) TO CHANGE A NON-ZERO NUMBER TO ZERO, TYPE "999."

$ SETUP
EXECUTION BEGINS...

  ************************************************************************

    *  IMAX   * JMAX/2  * KLIMIT  * MLIMIT  * KEXTRA  * MEXTRA  *
         60.       13.      250.      30.       5.        2.
         41.      999.                 1.

    *  ALPHA  *  SIZE   *  EPS.    * RHO-INF * LAMBDA  *  BETA   *
       2.000      3.000   0.000500   1.1760    6941.0   0.000201
                  2.       .001
```

```
    *  MODE   *  SYM.   * POINTS  *  LINES  * DIAGNOS * STD.DEV *
)       3.        1.       31.        9.       0.0      0.0
        1.      100.       21.        1.

    * PHIZERO * DELPHI  * YPZERO  * YPRANGE * XPZERO  * XPRANGE *
        0.0     22.500      0.0       0.0       0.0       1.500
                                                        1.

    * TST FUN * ADD FUN * GARRAY  *  GRAPH  * LIN PRT * MAP BND *
)      60.        0.0       1.        0.0       1.        0.0
        5.                 999.

    *    A    *    B    *    C    *    D    *    E    *    P    *
         1.000     0.700     0.900     0.071     0.071     0.0
                   1.        1.        999.      999.

    *    S    *    T    *    U    *    V    *    W    *    Q    *
         0.0       1.000     1.000     0.0       0.0       1.000
```



```
 ********************************************************************************
IHC0021 STOP       0
ERASE INPUT DATA *
SPLIT FILE FT04F001 INPUT DATA 1 EOF
EOF REACHED
FILE MODIFIED

DO YOU WISH THE TEST DATA READ IN?
no

WILL ADD-ON FUNCTION DATA BE READ IN? (EQ'N. NO. 8)
no

EXECUTE THE PROGRAM?
no
R; T=0.97/2.82 15.32.16
```

```
CFFLINE READ        COMPILE  EXEC

&TYPECUT ON NOTIME
FORTRAN SETUP
FORTRAN HOLOVERT
```

```
CFFLINE READ       HOLOFER  EXEC

&TYPECUT ON NOTIME                                                    HOLOC010
&IF &$ EQ RESTART &SKIP 1                                             HOLOC015
ERASE FILE FTC3F001 *                                                 HOLOC020
ERASE FILE FTC4F001 *                                                 HOLOC030
SPLIT INPUT DATA FILE FTC4F001 1 EOF                                  HOLOC040
&IF &$ EQ GO &GOTO -10                                                HOLOC050
&SP 1                                                                 HOLOC060
&IF &INDEX EQ 0 &PR ANSWER YES OR NO...                               HOLOC070
&IF &$ EQ INFO &INFO = INFO                                           HOLOC080
&IF &$ EQ NOINFO &INFO = NOINFO                                       HOLOC090
&IF &$ EQ FILES &FILES = FILES                                        HOLOC100
&IF &$ EQ NOFILES &FILES = NOFILES                                    HOLOC110
&IF &$ EQ INPUT &INPUT = INPUT                                        HOLOC120
&IF &$ EQ NOINPUT &INPUT = NOINPUT                                    HOLOC130
&IF &INFO EQ INFO &GOTO -1                                            HOLOC140
&IF &INFO EQ NOINFO &GOTO -2                                          HOLOC150
&SP 1                                                                 HOLOC160
&PRINT DO YOU WISH INFO TYPECUT?                                      HOLOC170
&READ ARGS                                                            HOLOC180
&IF &1 EQ NO &GOTO -2                                                 HOLOC190
-1 &TYPECUT OFF                                                       HOLOC200
EXEC INFO                                                             HOLOC210
&TYPECUT ON NOTIME                                                    HOLOC220
-2 &IF &INPUT EQ NOINPUT &GOTO -5                                     HOLOC230
&IF &INPUT EQ INPUT &GOTO -4                                          HOLOC240
&SP 1                                                                 HOLOC250
&PR DO YOU WISH INPUT DATA TYPED OUT OR CHANGED?                      HOLOC260
&READ ARGS                                                            HOLOC270
&IF &1 EQ NO &GOTO -5                                                 HOLOC280
&SP 1                                                                 HOLOC290
&PR THE INPUT VALUES FOLLOW:                                          HOLOC300
&PR --1) IF CHANGES ARE DESIRED TYPE NEW VALUE(S) UNDER OLD           HOLOC310
&PR --2) IF NO CHANGES, HIT SPACE BAR AND RETURN                      HOLOC320
&PR --3) TO CHANGE A NON-ZERO NUMBER TO ZERO, TYPE "999."             HOLOC330
&SP 1                                                                 HOLOC340
-4 & SETUP                                                            HOLOC350
ERASE INPUT DATA *                                                    HOLOC360
SPLIT FILE FTC4F001 INPUT DATA 1 EOF                                  HOLOC370
```

```
-5 &IF &FILES EQ NOFILES &GOTO -9                                              FOLOC380
&SP 1                                                                          FOLOC390
&PRINT DO YOU WISH THE TEST DATA READ IN?                                      FOLOC400
&READ ARGS                                                                     FOLOC410
&IF &1 EQ NO &GOTO -8                                                          FOLOC420
&SP 1                                                                          FOLOC430
&PR TYPE THE FILENAME AND FILETYPE OF THE DATA DECK TO BE USED.                FOLOC440
&READ ARGS                                                                     FOLOC450
&DATA = YES                                                                    FOLOC460
&FNAME = &1                                                                    FOLOC470
&FTYPE = &2                                                                    FOLOC480
ERASE FILE FTC1F001 *                                                          FOLOC490
SPLIT &1 &2 FILE FT01F001 1 EOF                                                FOLOC500
-9 &TYPEOUT ON NOTIME                                                          FOLOC510
&SP 1                                                                          FOLOC520
&PR WILL ADD-ON FUNCTION DATA BE READ IN? (EQ'N. NO. 8)                        FOLOC530
&READ ARGS                                                                     FOLOC540
&IF &1 EQ NO &GOTO -9                                                          FOLOC550
&SP 1                                                                          FOLOC560
&PR TYPE FILENAME AND FILETYPE OF ADD-ON FUNCTION DATA.                        FOLOC570
&READ ARGS                                                                     FOLOC580
&ADD = YES                                                                     FOLOC590
&FNAMA = &1                                                                    FOLOC600
&FTYPA = &2                                                                    FOLOC610
ERASE FILE FTC2F001 *                                                          FOLOC620
SPLIT &1 &2 FILE FT02F001 1 EOF                                                FOLOC630
-9 &SP 1                                                                       FOLOC640
&PR EXECUTE THE PROGRAM?                                                       FOLOC650
&READ ARGS                                                                     FOLOC660
&IF &1 EQ NO &GOTO -11                                                         FOLOC670
-10 &TYPEOUT OFF                                                               FOLOC680
GLOBAL LOADER TXTLIB SYSLIB SSPLIB                                             FOLOC690
&TYPEOUT ON NOTIME                                                             FOLOC700
* SAVE                                                                         FOLOC710
&IF &FNAME NE DATA &GOTO -11                                                   FOLOC720
&SP 1                                                                          FOLOC730
ALTER FILE FTC7F001 * CARRAY &FTYPE *                                          FOLOC740
-11 ERASE FILE FT04F001 *                                                      FOLOC750

CMSLINE READ        INFO EXEC

&TYPEOUT ON NOTIME
&SP 1
&PRINT SORRY ABOUT THAT, INFO NOT ENTERED, SEE APP. C, PART 1...
&SP 1
```

```
OFFLINE READ       SETUP    FORTRAN
C    SETUP IS THE PROGRAM WHICH READS THE INPUT PARAMATERS FOR THE RUN.   SETOC02C
      DIMENSION AR(42,2)                                                 SETOC040
      DATA ST/1H*/                                                       SETOC050
      READ  (4,59) (AR(I,1),I=1,42)                                      SETOC060
      DO  1 I=1,42                                                       SETOC070
1     AR(I,2)=C.                                                         SETOC080
      WRITE (6,50) (ST,I=1,75)                                           SETOC090
      WRITE (6,51) (AR(I,1),I= 1, 6)                                     SETOC100
      READ  (5,52) (AR(I,2),I= 1, 6)                                     SETOC110
      WRITE (6,53) (AR(I,1),I= 7,12)                                     SETOC120
      READ  (5,52) (AR(I,2),I= 7,12)                                     SETOC130
      WRITE (6,54) (AR(I,1),I=13,18)                                     SETOC140
      READ  (5,52) (AR(I,2),I=13,18)                                     SETOC150
      WRITE (6,60) (AR(I,1),I=19,24)                                     SETOC160
      READ  (5,52) (AR(I,2),I=19,24)                                     SETOC170
      WRITE (6,55) (AR(I,1),I=25,30)                                     SETOC180
      READ  (5,52) (AR(I,2),I=25,30)                                     SETOC190
      WRITE (6,56) (AR(I,1),I=31,36)                                     SETOC200
      READ  (5,52) (AR(I,2),I=31,36)                                     SETOC210
      WRITE (6,57) (AR(I,1),I=37,42)                                     SETOC220
      READ  (5,52) (AR(I,2),I=37,42)                                     SETOC230
      WRITE (6,58) (ST,I=1,75)                                           SETOC240
      DO  2 I=1,42                                                       SETOC250
      IF ((AR(I,2).NE.C.).AND.(AR(I,2).NE.999.)) AR(I,1)=AR(I,2)         SETOC260
2     IF (AR(I,2).EQ.999.) AR(I,1)=0.                                    SETOC270
      REWIND 4                                                           SETOC280
      WRITE (4,59) (AR(I,1),I=1,42)                                      SETOC290
50    FORMAT (/3X,75A1)                                                  SETOC300
51    FORMAT (/5X,'*  IMAX   * JMAX/2  * KLIMIT  * MLIMIT  * KEXTRA  *',SETOC310
     1' MEXTRA  *'/3X,6F10.0)                                            SETOC320
52    FORMAT (5X,6F10.3)                                                 SETOC330
53    FORMAT (/5X,'*  ALPHA  *  SIZE   *  EPS.   * RHO-INF * LAMBDA  *',SETOC340
     1'  BETA   *'/3X,2F10.3,F11.6,2X,F8.7,F9.1,F11.6)                   SETOC350
54    FORMAT (/5X,'*  MODE   *  SYM.   * POINTS  *  LINES  * DIAGNOS *',SETOC360
     1' STD.DEV *'/2X,5F10.0,F10.3)                                      SETOC370
60    FORMAT (/5X,'* PHIZERO * DELPHI  * YPZERO  * YPRANGE * XPZERO  *',SETOC380
     1' XPRANGE *'/4X,6F10.3)                                            SETOC390
55    FORMAT (/5X,'* TST FUN * ADD FUN * GARRAY  *  GRAPH  * LIN PRT *',SETOC400
     1' MAP BND *'/2X,6F10.0)                                            SETOC410
56    FORMAT (/5X,'*    A    *    B    *    C    *    D    *    E    *',SETOC420
     1'    P    *'/4X,6F10.3)                                           SETOC430
57    FORMAT (/5X,'*    S    *    T    *    U    *    V    *    W    *',SETOC440
     1'    Q    *'/4X,6F10.3)                                           SETOC450
58    FORMAT (/3X,75A1)                                                  SETOC460
59    FORMAT (6F12.7)                                                    SETOC470
      STOP                                                               SETOC480
      END                                                                SETOC490
```

```
CFFLINE READ       HOLOVERT FORTRAN                                          CALOOO10
C                                                                            CALOOO20
C    ***************************HOLOVERT*********************************   CALOOO30
C                                                                            CALOOO40
C    HOLOVERT INVERTS THE  FIELD AT AN ARRAY OF POINTS IN A VARIABLE         CALOOO50
C    COORDINATE SYSTEM.  IT SURVEYS NPTS POINTS EACH FOR A SET OF NLINS      CALOOO60
C    LINES ACROSS THE FIELD.                                                 CALOOO70
C                                                                            CALOOO80
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z        CALOOO90
      COMMON /TAB/ INDEX,KEXTRA,MEXTRA,KLIMIT,MLIMIT,KOUT,MOUT              CALOO100
      COMMON /TAB2/ IPT,KPT,LPT,BND,NPTS,NLINS,RHOINF,RLAMDA,BETA            CALOO110
      COMMON /OUT/ XP,THEO,CALC,ERR,RHO,CA,FA                                CALOO120
      COMMON /EQPARA/ A,B,C,D,E,P,Q,S,T,U,V,W,RO,RA,NC,NA,NCF,NAF            CALOO130
      COMMON /SYM/ ISYM,JSYM,MSYM,ECU,IMS,JMS,OSYM                           CALOO140
      COMMON /IC/ CMS,IN1,IN2,IN4                                            CALOO150
      DATA BL,PL,ST,EX,OH,SC,DH,BR/1H ,1H+,1H*,1HX,1HO,1H:,1H-,1H|/          CALOO160
      DIMENSION BB(7),TL(62),RO(101),RA(101)                                 CALOO170
C-----------------------------------------------------------------------     CALOO180
      DIMENSION G(5151),GA(5151),H(202,5),SCF(73,6),PDA(4000)                CALOO190
      DIMENSION THEO(51,11),CALC(51,11),ERR(51),RHO(51)                      CALOO200
      DIMENSION CA(51),FA(51,11),AR(42),XP(51),YP(51)                        CALOO210
      CMS=1.                                                                 CALOO220
C-----------------------------------------------------------------------     CALOO230
      IN1=5                                                                  CALOO240
      IN2=5                                                                  CALOO250
      IN4=5                                                                  CALOO260
      IF (CMS.NE.1.) GO TO 20                                                CALOO270
      IN1=1                                                                  CALOO280
      IN2=2                                                                  CALOO290
      IN4=4                                                                  CALOO300
      IF (CMS.EQ.1.) REWIND 4                                                CALOO310
20    READ (IN4,89) (AR(I),I=1,42)                                           CALOO320
      IMAX=AR(1)                                                             CALOO330
      JMAX=AR(2)*2.                                                          CALOO340
      IF (JMAX.LE.0) JMAX=1                                                  CALOO350
      KLIMIT=AR(3)                                                           CALOO360
      MLIMIT=AR(4)                                                           CALOO370
      KEXTRA=AR(5)                                                           CALOO380
      MEXTRA=AR(6)                                                           CALOO390
      JSYM=AR(14)                                                            CALOO400
      ALPHA=AR(7)                                                            CALOO410
      SIZE=AR(8)                                                             CALOO420
      EPS=AR(9)                                                              CALOO430
      MODE=AR(13)                                                            CALOO440
      DGN=AR(17)                                                             CALOO450
      RHOINF=AR(10)                                                          CALOO460
      RLAMDA=AR(11)*1.E-8                                                    CALOO470
      BETA=AR(12)
```

```
      NPTS=AR(15)                                                       CALCC480
      NLINS=AR(16)                                                      CALCC490
      SD=AR(18)                                                         CALCC500
      PHIZ=AR(19)                                                       CALCC510
      DELPHI=AR(20)                                                     CALCC520
      YPZERO=AR(21)                                                     CALCC530
      YPRNG=AR(22)                                                      CALCC540
      XPZERO=AR(23)                                                     CALCC550
      XPRNG=AR(24)                                                      CALCC560
      NOF=AR(25)                                                        CALCC570
      NAF=AR(26)                                                        CALCC580
      IPT=AR(27)                                                        CALCC590
      KPT=AR(28)                                                        CALCC600
      LPT=AR(29)                                                        CALCC610
      BND=AR(30)                                                        CALCC620
      A=AR(31)                                                          CALCC630
      B=AR(32)                                                          CALCC640
      C=AR(33)                                                          CALCC650
      D=AR(34)                                                          CALCC660
      E=AR(35)                                                          CALCC670
      P=AR(36)                                                          CALCC680
      S=AR(37)                                                          CALCC690
      T=AR(38)                                                          CALCC700
      U=AR(39)                                                          CALCC710
      V=AR(40)                                                          CALCC720
      W=AR(41)                                                          CALCC730
      Q=AR(42)                                                          CALCC740
      IF (DGN.GE.4) WRITE (6,89) (AR(I),I=1,42)                         CALCC750
      NNN=2                                                             CALCC760
      IF (MODE.LT.0) NNN=1                                              CALCC770
      IF (MODE.GT.5) NNN=3                                              CALCC780
      IF (MODE.GT.5) MODE=MODE-10                                       CALCC790
      NGP=0                                                             CALCC800
      IF (KLIMIT.LT.KEXTRA) KEXTRA=KLIMIT                               CALCC810
      IF (MLIMIT.LT.MEXTRA) MEXTRA=MLIMIT                               CALCC820
      IF (IPT.LT.0) NGP=IPT                                             CALCC830
      IF (IPT.LT.0) IPT=-IPT                                            CALCC840
      ISYM=2.1-(FLOAT(JSYM)/2.-FLOAT(JSYM/2))*2                         CALCC850
      IF (JSYM.EQ.0) ISYM=1                                             CALCC860
      IF (JSYM.GT.JMAX) ISYM=2                                          CALCC870
      IF (ISYM.EQ.1) JMAX=((JMAX+1)/2)*2                                CALCC880
      RJMX=JMAX                                                         CALCC890
      MSYM=JSYM                                                         CALCC900
      IF ((MSYM.EQ.0).OR.(MSYM.GT.JMAX)) MSYM=1                         CALCC910
      FCU=ISYM*JSYM*JMAX                                                CALCC920
      IF ((JSYM.GT.JMAX).OR.(JSYM.EQ.0)) FCU=JMAX                       CALCC930
      QSYM=FCU/RJMX                                                     CALCC940
      IMS=(IMAX+ISYM-1)/ISYM                                            CALCC950
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      JMS=JMAX                                                          CALCC960
      IF (ISYM.EQ.1) JMS=(JMAX/2+1)/2                                   CALCC970
      IF (JSYM.EQ.0) JMS=JMAX/2                                         CALCC980
      MODE=ABS(AR(13))                                                  CALCC990
      XO=0.                                                             CALC1000
      YO=0.                                                             CALC1010
      ZD=0.                                                             CALC1020
      PHISYM=0.                                                         CALC1030
      RS=SIZE/2.                                                        CALC1040
      RHOS=1.2E6                                                        CALC1050
      BOX=RHOINF*BETA/RHOS/RLAMDA                                       CALC1060
      RPTS=NPTS                                                         CALC1070
      XPR=0.                                                            CALC1080
      IF (NPTS.GT.1) XPR=XPRNG/(RPTS-1.)/2.                             CALC1090
      XPM=-XPR                                                          CALC1100
      PIE=3.14159265358979                                              CALC1110
      MONE=1                                                            CALC1120
        WRITE (6,58) IMAX,JMAX,IMS,JMS,ISYM,JSYM,MSYM,QSYM,ECU          CALC1130
   58   FORMAT (3X,'IMAX,JMAX,IMS,JMS,ISYM,JSYM,MSYM,QSYM,ECU',/        CALC1140
     1    7I5,2F7.3/)                                                  CALC1150
      NTWO=2                                                            CALC1160
      IX=IMAX+JMAX                                                      CALC1170
      NF=IN1                                                            CALC1180
      IF ((MODE.EQ.1).AND.(NOF.EQ.8).AND.(DGN.GE.1.)) WRITE (6,69)      CALC1190
      IF ((MODE.EQ.1).AND.(NOF.EQ.8)) CALL FREAD (NO,RO,NF,ZD)          CALC1200
      Z=ZD                                                              CALC1210
      IF (DGN.GE.1.) WRITE (6,68)                                       CALC1220
      CALL GARRAY (G,GA,NOF,DGN,MONE,XO,YO,PHISYM)                      CALC1230
      LM=1                                                              CALC1240
      IF ((LPT.EQ.0).AND.(BND.EQ.0)) LM=0                               CALC1250
      IIMX=IMAX+1                                                       CALC1260
      JJMX=JMAX+1                                                       CALC1270
      IJMX=IMAX*JMAX                                                    CALC1280
      NBD=1                                                             CALC1290
      IF (JSYM.EQ.0) NBD=2                                              CALC1300
      KBD=KLIMIT*NBD                                                    CALC1310
      DO 15 IJ=1,IJMX                                                   CALC1320
   15 GA(IJ)=0.                                                         CALC1330
      IF (NAF.EQ.0) GO TO 16                                            CALC1340
      NF=IN2                                                            CALC1350
      IF ((NAF.EQ.8).AND.(DGN.GE.1.)) WRITE (6,69)                      CALC1360
      IF (NAF.EQ.8) CALL FREAD (NA,RA,NF,ZD)                            CALC1370
      MST=MODE                                                          CALC1380
      MODE=1                                                            CALC1390
      IF (DGN.GE.1.) WRITE (6,68)                                       CALC1400
      IF (NAF.NE.0) CALL GARRAY (GA,G,NAF,DGN,NTWO,XO,YO,PHISYM)        CALC1410
      MODE=MST                                                          CALC1420
      DO 6 IJ=1,IJMX                                                    CALC1430
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6       G(IJ)=G(IJ)+GA(IJ)                                              CALC1440
16      RLINS=NLINS                                                     CALC1450
        IF (NAF.EQ.8) WRITE (6,88) NA,(RA(L),L=1,NA)                    CALC1460
        IF (NOF.EQ.8) WRITE (6,87) NO,(RO(I),I=1,NO)                    CALC1470
        IF (LM.EQ.0) GO TO 14                                           CALC1480
        RB(1)=-1.                                                       CALC1490
        DO 1 I=2,7                                                      CALC1500
1       RB(I)=RB(I-1)+.5                                                CALC1510
        TPIE=2.*PIE                                                     CALC1520
        MPIE=-PIE                                                       CALC1530
        DYP=0.                                                          CALC1540
        DXP=0.                                                          CALC1550
        IF (NLINS.GT.1) DYP=YPRNG/(RLINS-1.)                            CALC1560
        IF (NPTS.GT.1) DXP=XPRNG/(RPTS-1.)                              CALC1570
        IF ((DGN.GE.1.).AND.(NNN.EQ.2)) WRITE (6,64)                    CALC1580
        IF(NNN.EQ.2) CALL BDGEN (G,H,SCF,DGN,NBD,BDA,KBD)               CALC1590
        DO 5 J=1,NLINS                                                  CALC1600
        IF (DGN.GE.1.) WRITE (6,67) J                                   CALC1610
        RJM=J-1                                                         CALC1620
        PHI=PHIZ+DELPHI*RJM                                             CALC1630
        YP(J)=YPZERO+DYP*RJM                                            CALC1640
        PSI=(PHI+90.)*PIE/180.                                          CALC1650
        TAU=PSI-PHISYM                                                  CALC1660
        IF (LPT.EQ.0) GO TO 9                                           CALC1670
        IF (LPT.LE.1) WRITE (6,78) (ST,I=1,124)                         CALC1680
        IF (LPT.GT.1) WRITE (6,74) (ST,I=1,95)                          CALC1690
        IF ((CMS.EQ.1.).AND.(LPT.GT.1)) READ (5,79) ZZ                  CALC1700
        WRITE (6,86)                                                    CALC1710
        WRITE (6,85) Z,PHI,YP(J)                                        CALC1720
        WRITE (6,76)                                                    CALC1730
        IF (MODE.EQ.1) WRITE (6,83) (RB(I),I=1,7)                       CALC1740
        IF (MODE.GT.1) WRITE (6,80) (RB(I),I=1,7)                       CALC1750
        WRITE (6,81) (DH,I=1,54),(PL,II=1,13)                           CALC1760
9       IC=0                                                            CALC1770
        DO 3 I=1,NPTS                                                   CALC1780
        RIM=I-1                                                         CALC1790
        THETO(I,J)=0.                                                   CALC1800
        CA(I)=0.                                                        CALC1810
        FA(I,J)=0.                                                      CALC1820
        ERR(I)=0.                                                       CALC1830
        CALC(I,J)=0.                                                    CALC1840
        RHO(I)=0.                                                       CALC1850
        XP(I)=XPZERO+DXP*RIM                                            CALC1860
        XPII=ABS(XP(I))                                                 CALC1870
        IF (XPII.LT.1.E-10) XP(I)=0.                                    CALC1880
        RS=SQRT(XP(I)**2+YP(J)**2)/HS                                   CALC1890
        IF (RS.GT.1.) GO TO 13                                          CALC1900
        THT=ATAN2(YP(J),XP(I))                                          CALC1910
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      IF (XPII.EQ.0.) THT=0.                                                    CALC1920
      SIG=TAL-PIE/2.+THT                                                        CALC1930
      IF (SIG.GT.PIE) SIG=SIG-TPIE                                              CALC1940
      IF (SIG.LT.MPIE) SIG=SIG+TPIE                                             CALC1950
      SIGI=SIG                                                                  CALC1960
      XS=RS*COS(SIG)                                                            CALC1970
      IF (DGN.GE.1.) WRITE (6,44) SIG                                           CALC1980
44    FORMAT ('   SIG=',E10.3)                                                  CALC1990
      YS=RS*SIN(SIG)                                                            CALC2000
      IF(DGN.GE.5) WRITE (6,57) PHI,DELPHI,PSI,TAU,THT,SIG,SIGI,XS,YS           CALC2010
57      FORMAT ('  ANGLES=',10E10.3)                                          CALC2020
      RI=I                                                                      CALC2030
      F=0.                                                                      CALC2040
      IF (DGN.GE.2.) WRITE (6,65) I                                             CALC2050
      CALL FUNCT (XS,YS,FA(I,J),NAF,DGN,NTWO)                                   CALC2060
      IF (MODE.EQ.1) CALL FUNCT (XS,YS,F,NCF,DGN,MONE)                          CALC2070
      THEO(I,J)=F                                                               CALC2080
      IF (NNN.GE.2) REWIND 3                                                    CALC2090
      IF(NNN.GE.2) CALL FIELD (RS,SIGI,SOLN,NBD,BDA,DGN,KBD)                    CALC2100
      IF(NNN.EQ.1) CALL FIELD2 (RS,SIGI,SOLN,G,H,SCF,DGN)                       CALC2110
      CA(I)=SOLN/BOX/HS                                                         CALC2120
      CALC(I,J)=CA(I)-FA(I,J)                                                   CALC2130
      RHO(I)=RHOINF*(CALC(I,J)+1.)                                              CALC2140
      ERR(I)=CA(I)                                                              CALC2150
      IF (MODE.EQ.1) ERR(I)=(CALC(I,J)-THEO(I,J))                               CALC2160
      IF (MODE.GT.1) THEO(I,J)=FA(I,J)                                          CALC2170
      IF (LPT.EQ.0) GO TO 3                                                     CALC2180
13    LC=0                                                                      CALC2190
      TL(1)=PL                                                                  CALC2200
      TTL=0.                                                                    CALC2210
      IF ((XP(I).GT.XPM).AND.(XP(I).LT.XPR)) TTL=1.                             CALC2220
      IF (IC.EQ.5) IC=0                                                         CALC2230
      IF (IC.EQ.0) TL(1)=PL                                                     CALC2240
      DO 2 L=2,62                                                               CALC2250
      TL(L)=BL                                                                  CALC2260
      IF ((I.EQ.1).OR.(TTL.EQ.1.).OR.(I.EQ.NPTS)) TL(L)=PL                      CALC2270
      IF (LC.EQ.10) LC=0                                                        CALC2280
      IF ((IC.EQ.0).AND.(LC.EQ.0)) TL(L)=PL                                     CALC2290
2     LC=LC+1                                                                   CALC2300
      TL(2)=PL                                                                  CALC2310
      TL(22)=PL                                                                 CALC2320
      TL(62)=PL                                                                 CALC2330
      IC=IC+1                                                                   CALC2340
      RLW=(CA(I)+1.)*20.+2.5                                                    CALC2350
      LW=RLW                                                                    CALC2360
      IF (LW.GT.62) LW=62                                                       CALC2370
      IF (LW.LT.2) LW=2                                                         CALC2380
      TL(LW)=SC                                                                 CALC2390
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      RLY=(FA(I,J)+1.)*20.+2.5                                          CALC24CC
      LY=RLY                                                            CALC2410
      IF (LY.GT.62) LY=62                                               CALC2420
      IF (LY.LT.2) LY=2                                                 CALC2430
      IF (NAF.NE.C) TL(LY)=ST                                           CALC2440
      RLX=(THEC(I,J)+1.)*20.+2.5                                        CALC2450
      LX=RLX                                                            CALC2460
      IF (LX.GT.62) LX=62                                               CALC2470
      IF (LX.LT.2) LX=2                                                 CALC2480
      IF (MODE.EQ.1) TL(LX)=CH                                          CALC2490
      RLZ=(CALC(I,J)+1.)*20.+2.5                                        CALC2500
      LZ=RLZ                                                            CALC2510
      IF (LZ.GT.62) LZ=62                                               CALC2520
      IF (LZ.LT.2) LZ=2                                                 CALC2530
      TL(LZ)=EX                                                         CALC2540
      WRITE (6,82)MOUT,KOUT,INDEX,THEO(I,J),ERR(I),CALC(I,J),RHO(I),   CALC2550
     1XP(I),(TL(L),L=1,62)                                              CALC2560
      IF ((NPTS.LE.20).AND.(I.NE.NPTS)) WRITE (6,79)                    CALC2570
3     CONTINUE                                                          CALC2580
      IF (LPT.NE.0) WRITE (6,81) (DH,I=1,54),(PL,I=1,13)                CALC2590
      TMAX=0.                                                           CALC2600
      TMIN=0.                                                           CALC2610
      IE=0                                                              CALC2620
      BE=0.                                                             CALC2630
      ER=0.                                                             CALC2640
      DO 4 I=1,NPTS                                                     CALC2650
      TH=THEC(I,J)                                                      CALC2660
      IF (TH.GT.TMAX) TMAX=TH                                           CALC2670
      IF (TH.LT.TMIN) TMIN=TH                                           CALC2680
      ER=ABS(CALC(I,J)-TH)                                              CALC2690
      IF (ER.LE.BE) GO TO 4                                             CALC2700
      BE=ER                                                             CALC2710
      IE=I                                                              CALC2720
4     CONTINUE                                                          CALC2730
      TMM=TMAX-TMIN                                                     CALC2740
      EB=RHOINF*(CALC(IE,J)-THEO(IE,J))                                 CALC2750
      IF (TMM.NE.0.) BE=(CALC(IE,J)-THEO(IE,J))*100./TMM                CALC2760
      IF ((MODE.EQ.1).AND.(LPT.NE.0)) WRITE (6,75) EB,XP(IE),BE         CALC2770
      IF (DELPHI.NE.0.) YP(J)=PHI                                       CALC2780
5     CONTINUE                                                          CALC2790
      IF (BND.EQ.0.) GO TO 14                                           CALC2800
      IF (LPT.EQ.1) WRITE (6,78) (ST,I=1,124)                           CALC2810
      IF (LPT.GT.1) WRITE (6,74) (ST,I=1,95)                            CALC2820
      IF ((CMS.EQ.1.).AND.(LPT.GT.1)) READ (5,79) ZZ                    CALC2830
      IF (DGN.GE.1.) WRITE (6,63)                                       CALC2840
      CALL MAP (NPTS,NLINS,CALC,NCF,7,BND)                              CALC2850
14    IF (NAF.EQ.0) GO TO 10                                            CALC2860
      NAC=10*NCF+NAF                                                    CALC2870
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      IF ((DGN.GE.1.).AND.(NGP.EQ.-3)) WRITE (6,62)                     CALC2880
      IF (NGP.EQ.-3) CALL GPUNCH (Z,XC,YC,PHISYM,NAO,IMAX,JMAX,G)       CALC2890
      DO 7 IJ=1,IJMX                                                    CALC2900
7     G(IJ)=G(IJ)-GA(IJ)                                                CALC2910
10    IF (IPT.LE.0) GO TO 11                                            CALC2920
      IF ((IPT.EQ.1).OR.(IPT.EQ.3)) WRITE (6,73) (ST,I=1,124)           CALC2930
      IF ((IPT.EQ.2).OR.(IPT.GE.4)) WRITE (6,74) (ST,I=1,95)            CALC2940
      IF ((CMS.EQ.1.).AND.((IPT.EQ.2).OR.(IPT.GE.4))) READ (5,79) ZZ    CALC2950
      CALL GPRINT (G,MONE)                                              CALC2960
      IF (NGP.EQ.-1) CALL GPUNCH (Z,XC,YC,PHISYM,NOE,IMAX,JMAX,G)       CALC2970
      IF (IPT.EQ.3) WRITE (6,78) (ST,I=1,124)                           CALC2980
      IF (IPT.GE.4) WRITE (6,74) (ST,I=1,95)                            CALC2990
      IF ((CMS.EQ.1.).AND.(IPT.GE.4)) READ (5,79) ZZ                    CALC3000
      IF (IPT.GE.3) CALL GPRINT (GA,NTWO)                               CALC3010
11    IF (KPT.LE.0) GO TO 12                                            CALC3020
      IF ((KPT.EQ.1).OR.(KPT.EQ.3)) WRITE (6,78) (ST,I=1,124)           CALC3030
      IF ((KPT.EQ.2).OR.(KPT.GE.4)) WRITE (6,74) (ST,I=1,95)            CALC3040
      IF ((CMS.EQ.1.).AND.((KPT.EQ.2).OR.(KPT.GE.4))) READ (5,79) ZZ    CALC3050
      IF (DGN.GE.1.) WRITE (6,61)                                       CALC3060
      CALL GPLOT (G,GA,JMS)                                             CALC3070
12    WRITE (6,78) (EX,I=1,124)                                         CALC3080
      AGAIN=ST                                                          CALC3090
      IF (CMS.NE.1.) READ(5,60) AGAIN                                   CALC3100
      IF (AGAIN.EQ.BL) GO TO 20                                         CALC3110
      WRITE (6,77)                                                      CALC3120
89    FORMAT (6F12.7)                                                   CALC3130
89    FORMAT (/'      THE INPUT DATA FOR ADD-ON FUNCTION NO.8 (',I3,    CALC3140
     1' POINTS) WAS:'/7(11F10.3/))                                      CALC3150
87    FORMAT (/'      THE INPUT DATA FOR THE TEST FUNCTION NO.8, (',I3, CALC3160
     1' POINTS) WAS:'/7(13F10.3/))                                      CALC3170
86    FORMAT (1H1//'    THE INVERTED CROSS SECTION FOR:')               CALC3180
85    FORMAT (10X,'Z  =',F8.3,' CM.'/10X,'PHI=',F8.3,' DEGREES'/10X,    CALC3190
     14HY' =,F8.3,' CM.',44X,'O = ORIGINAL FUNCTION'/                   CALC3200
     27CX,'* = ADD-ON FUNCTION')                                        CALC3210
84    FORMAT (' ADJUST PAGE, HIT SPACE AND RETURN.')                    CALC3220
83    FORMAT ('   LIMIT  MAX  ORIGINAL     ABS.  COMPUTED (MG/CC)'/'    MCALC3230
     1   K TERM  FUNCTION    ERROR  FUNCTION DENSITY ',6H  X'  ,F4.1,   CALC3240
     26F10.1)                                                           CALC3250
82    FORMAT (2X,I2,1X,I3,1X,I4,1X,F9.4,2X,F7.3,1X,F9.4,1X,F7.3,        CALC3260
     1F7.3,1X,62A1)                                                     CALC3270
81    FORMAT (3X,54A1,2X,A1,12(4X,A1))                                  CALC3280
80    FORMAT ('   LIMIT  MAX',3X,'ADD-ON',6X,'THE',4X,'DENSITY (MG/CC)'/CALC3290
     1'    M   K TERM',2X,'FUNCTION',5X,'SUM',3X,'FUNCTION DENSITY ',  CALC3300
     26F   X'  ,F4.1,6F10.1)                                            CALC3310
79    FORMAT (1X,F10.3)                                                 CALC3320
78    FORMAT (1X,124A1)                                                 CALC3330
77    FORMAT (1X//)                                                     CALC3340
76    FORMAT (4X,'FUNCTION = (RHO/RHO-INFINITY)-1.0',33X,': = THE'      CALC3350
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     1,' INVERTED SUM'/7CX,'X = COMPUTED FUNCTION')                          CAL03360
75     FORMAT ('     LARGEST ERROR: ',F8.6,' GMS/CC;      AT ',3HX'=,F6.3CAL03370
     1,';',4X,F1C.2,' PERCENT.'//)                                           CAL03380
74     FORMAT (1X,47A1,' SET PAGE, HIT SPACE, RETURN ',48A1)                 CAL03390
69     FORMAT ('   CALL FREAD')                                             CAL03400
68     FORMAT ('   CALL GARRAY')                                            CAL03410
67     FORMAT ('   LINE',I3,' DO LOOP')                                     CAL03420
66     FORMAT ('   POINT',I3,' CALL FUNCT')                                 CAL03430
65     FORMAT ('   CALL FIELD')                                             CAL03440
64     FORMAT ('   CALL BDGEN')                                             CAL03450
63     FORMAT ('   CALL MAP')                                               CAL03460
62     FORMAT ('   CALL GPUNCH')                                            CAL03470
61     FORMAT ('   CALL GPLOT')                                             CAL03480
60     FORMAT (8CA1)                                                        CAL03490
       STOP                                                                 CAL03500
       END                                                                  CAL03510
                                                                            CAL03520
CCCCCC1                                                                     SUBCCC10
C                                                                           SUBCCC20


       SUBROUTINE BDGEN (G,H,SCF,DGN,NBD,BDA,KBD)                           SUBCCC30
C                                                                           SUBCCC40
C    BDGEN EVALUATES THE B AND D COEFFICIENTS FOR ALL M AND K, AND WRITES   SUBCCC5C
C    THE ARRAY LINEARLY ON DISK.                                            SUBCCC60
C                                                                           SUBCCC7C
       COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z      SUBCCC80
       COMMON /TAP/ INDEX,KEXTRA,MEXTRA,KLIMIT,MLIMIT,KOUT,MOUT             SUBCCC9C
       COMMON /SYM/ ISYM,JSYM,MSYM,FCL,IMS,JMS,QSYM                         SUBCC1CC
       DIMENSION G(IJMX),H(IIMX,5),SCF(JJMX,5),BDA(KBD)                     SUBCC110
C    INITIALIZE THE VALUES:                                                 SUBCC120
       INDEX=C                                                              SUBCC130
       KL2=NBD*KLIMIT                                                       SUBCC140
       REWIND 3                                                             SUBCC150
       JJMX6=JJMX*6                                                         SUBCC160
       IIMX2=(IIMX+1)/2                                                     SUBCC170
       PIE=3.1415926535897S3                                                SUBCC180
       RIMAX=IMAX                                                           SUBCC190
       KLMP=KLIMIT+1                                                        SUBCC200
       DX=2./RIMAX                                                          SUBCC210
       RJMAX=JMAX                                                           SUBCC220
       DXI=2.*PIE/FCU                                                       SUBCC230
C    INITIALIZE THE MODIFIED HERMITE POLYNOMIAL ARRAY:  VECTORS:            SUBCC24C
C    (1)=H1, (2)=H2, (3)=ALPHA*X(I), (4)=HM+2 STORED, (5)=HM+1 STORED       SUBCC250
       DO 1 II=1,IIMX2                                                      SUBCC260
       RII=II                                                               SUBCC270
       IIM=IIMX-II+1                                                        SUBCC280
       H(II,3)=ALPHA*(RII*DX-DX-1.)                                         SUBCC290
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      H(IIM,3)=-H(II,3)                                                    SUBCC300
      H(II,1)=2.*H(II,3)                                                   SUBCC310
      H(II,2)=(H(II,3)*H(II,1)-1.)/3.                                      SUBCC320
      H(IIM,1)=-H(II,1)                                                    SUBCC330
      H(IIM,2)=H(II,2)                                                     SUBCC340
      H(II,5)=H(II,2)                                                      SUBCC350
      H(IIM,5)=H(IIM,2)                                                    SUBCC360
      H(II,4)=H(II,1)                                                      SUBCC370
1     H(IIM,4)=H(IIM,1)                                                    SUBCC380
      SIGN=1.                                                              SUBCC390
C     INITIATE THE SIN/COS ARRAY:                                          SUBCC400
      DO 2 J=1,JJMX                                                        SUBCC410
      RJM=J-1                                                              SUBCC420
      SCF(J,1)=0.                                                          SUBCC430
      SCF(J,2)=1.                                                          SUBCC440
      SCF(J,3)=SIN(RJM*DXI-PIE/2.)                                         SUBCC450
      SCF(J,4)=COS(RJM*DXI-PIE/2.)                                         SUBCC460
      SCF(J,5)=0.                                                          SUBCC470
2     SCF(J,6)=0.                                                          SUBCC480
      MS=0                                                                 SUBCC490
C     COMMENCE THE M LOOP:                                                 SUBCC500
      DO 7 MP=1,MLIMIT                                                     SUBCC510
      M=MP-1                                                               SUBCC520
      RM=M                                                                 SUBCC530
      SIGN=-SIGN                                                           SUBCC540
      IF (DGN.LE.-4) WRITE (6,88) SCF(1,1),SCF(2,1),SCF(1,2),SCF(2,2)      SUBCC550
C     TEST FOR SYMMETRY SKIPS:                                             SUBCC560
      IF (MS.EQ.MSYM) MS=0                                                 SUBCC570
      TOTAL=0.                                                             SUBCC580
      MS=MS+1                                                              SUBCC590
      IF (MS.NE.1) GO TO 6                                                 SUBCC600
C     COMMENCE THE K LOOP:                                                 SUBCC610
      DO 5 KP=1,KLIMIT                                                     SUBCC620
      K=KP-1                                                               SUBCC630
      FK=KP                                                                SUBCC640
      RK=K                                                                 SUBCC650
      INDEX=INDEX+1                                                        SUBCC660
C     CALL THE B & D COEFFICIENTS AND WRITE THEM ON DISK:                  SUBCC670
      CALL BD (M,K,G,H,SCF,B,D,JJMX6)                                      SUBCC680
      IF (DGN.LE.-2) WRITE (6,89) M,K,B,D                                  SUBCC690
      IF (DGN.LE.-4) WRITE (6,88) H(1,1),H(1,2),H(1,4),H(1,5)              SUBCC700
      KK=K*NBD+1                                                           SUBCC710
      K2=KP*NBD                                                            SUBCC720
      BDA(K2)=D                                                            SUBCC730
      BDA(KK)=B                                                            SUBCC740
C     GENERATE THE NEXT ORDER OF THE SET OF HERMITE POLYNOMIALS FOR NEW K: SUBCC750
      CRDER=M+2*KP+1                                                       SUBCC760
      FA=SQRT(FK*(FK+RM))/CRDER                                            SUBCC770
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      HB=2.*SQRT((PK+1.)*(RM+PK+1.))/(ORDER+1.)/(ORDER+2.)                SUBCC78C
      DO 5 II=1,IIMX2                                                     SUBCC79C
      IIM=IIMX-II+1                                                       SUBCC80C
      H(II,1)=2.*(H(II,3)*H(II,2)-HA*H(II,1))                             SUBCC81C
      H(IIM,1)=SIGN*H(II,1)                                               SUBCC82C
5     H(II,2)=HB*(H(II,3)*H(II,1)-ORDER*H(II,2))                          SUBCC83C
C     ADVANCE THE SIN/COS ARRAY FOR THE NEXT M:                           SUBCC84C
      DO 3 J=1,JJMX                                                       SUBCC85C
      IF (DGN.LE.-5) WRITE (6,87) (SCF(J,NT),NT=1,6)                      SUBCC86C
87      FORMAT ('  SIN/COS MXI:'8E10.3)                                  SUBCC87C
      STEMP=SCF(J,1)                                                      SUBCC88C
      SCF(J,1)=SCF(J,1)*SCF(J,4)+SCF(J,2)*SCF(J,3)                        SUBCC89C
3     SCF(J,2)=SCF(J,2)*SCF(J,4)-STEMP*SCF(J,3)                           SUBCC90C
      DO 4 J=1,JMAX                                                       SUBCC91C
      SCF(J,5)=SCF(J+1,1)-SCF(J,1)                                        SUBCC92C
4     SCF(J,6)=SCF(J+1,2)-SCF(J,2)                                        SUBCC93C
      WRITE (3) (BDA(I),I=1,KBD)                                          SUBCC94C
      IF (DGN.LE.-3) WRITE (6,88) (BDA(I),I=1,10)                         SUBCC95C
      IF (JSYM.GT.JMAX) RETURN                                            SUBCC96C
6     RM=RM+1                                                             SUBCC97C
C     REGENERATE THE HERMITE ARRAY FOR NEW M, K=0:                        SUBCC98C
      DO 7 II=1,IIMX2                                                     SUBCC99C
      IIM=IIMX-II+1                                                       SUBC10CC
      H(II,2)=H(II,4)*SQRT(RM)/(RM+1.)                                    SUBC1010
      H(II,1)=H(II,5)*(RM+2.)                                             SUBC1020
      H(IIM,1)=-SIGN*H(II,1)                                              SUBC1030
      H(II,2)=2.*SQRT(RM+1)*(H(II,3)*H(II,1)-(RM+1.)*H(II,2))             SUBC1040
      H(II,2)=H(II,2)/(RM+2.)/(RM+3.)                                     SUBC1050
      H(II,4)=H(II,1)                                                     SUBC1060
7     H(II,5)=H(II,2)                                                     SUBC1070
89    FORMAT ('   M=',I4,',   K=',I4,',   B=',E10.4,',   D=',E10.4)       SUBC1080
88    FORMAT (2X,10E10.3)                                                 SUBC1090
      RETURN                                                              SUBC1100
      END                                                                 SUBC1110
CCCCCC2                                                                   SUBC1120
C                                                                         SUBC1130


      SUBROUTINE FIELD (RS,SIG,SCLN,NBD,BDA,DGN,KBD)                      SUBC1140
C                                                                         SUBC1150
C     FIELD EVALUATES THE VALUE OF THE FIELD FUNCTION AT A PARTICULAR     SUBC1160
C     POINT DESIGNATED IN CYLINDRICAL COORDINATES, BY USING THE INVERSION SUBC1170
C     EQUATION OF MALDONADO, ET.AL.  FIELD USES THE ARRAY OF  B & D       SUBC1180
C     COEFFICIENTS GENERATED IN SUBROUTINE BDGEN.                         SUBC1190
C                                                                         SUBC1200
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SC,IX,Z     SUBC1210
      COMMON /TAB/ INDEX,MEXTRA,MEXTRA,KLIMIT,NLIMIT,KOUT,MOUT           SUBC1220
      COMMON /SYM/ ISYM,JSYM,MSYM,ECU,IMS,JMS,QSYM                        SUBC1230
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      DIMENSION PDA(KBD), STK(52), STM(52)                              SUBC12400
C     INITIALIZE THE VALUES:                                            SUBC12500
      INDEX=0                                                           SUBC12600
      MTIMER=0                                                          SUBC12700
      KOUT=0                                                            SUBC12800
      MOUT=0                                                            SUBC12900
      MMAX=0                                                            SUBC13000
      KMAX=0                                                            SUBC13100
      TOTAL=0.                                                          SUBC13200
      JJMX6=JJMX*6                                                      SUBC13300
      REWIND 3                                                          SUBC13400
      AR=ALPHA*RS                                                       SUBC13500
      ARG=AR**2                                                         SUBC13600
      EXPON=EXP(-ARG)                                                   SUBC13700
      PIE=3.14159265358979                                              SUBC13800
      APP=ALPHA/PIE/PIE                                                 SUBC13900
      M=0                                                               SUBC14000
      RM=M                                                              SUBC14100
      RIMAX=IMAX                                                        SUBC14200
      DX=2./RIMAX                                                       SUBC14300
      RJMAX=JMAX                                                        SUBC14400
      SIGN=1.                                                           SUBC14500
      STK(1)=0.                                                         SUBC14600
      STM(1)=0.                                                         SUBC14700
      SMS=0.                                                            SUBC14800
      CMS=1.                                                            SUBC14900
      SMI=SIN(SIG)                                                      SUBC15000
      CMI=COS(SIG)                                                      SUBC15100
      MEP=MEXTRA+1                                                      SUBC15200
      DO 16 MB=1,MEP                                                    SUBC15300
16    STM(MB)=0.                                                        SUBC15400
      FM=1.                                                             SUBC15500
      MS=0                                                              SUBC15600
C     COMMENCE THE M LOOP:                                              SUBC15700
2     SIGN=-SIGN                                                        SUBC15800
      K=0                                                               SUBC15900
      RK=K                                                              SUBC16000
      FM=M                                                              SUBC16100
      ARM=1.                                                            SUBC16200
      IF (M.NE.0) ARM=AR**M                                             SUBC16300
      KTIMER=0                                                          SUBC16400
      KEP=KEXTRA+1                                                      SUBC16500
      DO 15 KB=1,KEP                                                    SUBC16600
15    STK(KB)=0.                                                        SUBC16700
      SIGNK=-1.                                                         SUBC16800
C     COMPUTE THE K=0 & K=1 ORDERS OF LAGUERRE POLYNOMIAL FOR GIVEN M:  SUBC16900
      FM=0.                                                             SUBC17000
      P=SQRT(1./FM)                                                     SUBC17100
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      PP=(RM+1.-ARG)*SQRT(1./FM/(RM+1.))                                SUBC1720
C     TEST FOR SYMMETRY SKIPS:                                          SUBC1730
      IF (MS.EQ.MSYM) MS=0                                              SUBC1740
      MS=MS+1                                                           SUBC1750
      IF (MS.NE.1) GO TO 7                                              SUBC1760
C     READ A LINE OF B & D COEFFICIENTS FOR GIVEN M:                    SUBC1770
      READ (3) (BDA(I),I=1,KBD)                                         SUBC1780
      IF (DGN.LE.-6) WRITE (6,88) (BDA(I),I=1,10)                       SUBC1790
C     COMMENCE THE K LOOP:                                              SUBC1800
3     INDEX=INDEX+1                                                     SUBC1810
      SIGNK=-SIGNK                                                      SUBC1820
C     COMPUTE THE M,K SUMMATION TERM:                                   SUBC1830
      KK=K*NBD+1                                                        SUBC1840
      B=BDA(KK)                                                         SUBC1850
      D=0.                                                              SUBC1860
      IF (NBD.EQ.2) D=BDA(KK+1)                                         SUBC1870
      BRAKET=B                                                          SUBC1880
      IF(RM.EQ.0.) GO TO 4                                              SUBC1890
      BRAKET=B*CMS+D*SMS                                                SUBC1900
4     ADD=SIGNK*BRAKET*P*ARM                                            SUBC1910
      TOTAL=TOTAL+ADD                                                   SUBC1920
      IF (DGN.GT.-5) GO TO 5                                            SUBC1930
      STOT=TOTAL*EXPON*APB/BOX/SIZE                                     SUBC1940
      WRITE (6,89) M,K,STOT,ADD,BRAKET,P,ARM,B,CMS,D,SMS                SUBC1950
C     ESTABLISH CHECK AS THE RELATIVE SIZE OF THE M,K TERM OF THE SERIES: SUBC1960
5     CHECK=ABS(ADD)                                                    SUBC1970
      IF (TOTAL.GT.EPS) CHECK=ABS(ADD/TOTAL)                            SUBC1980
C     ADVANCE THE K INDEX:                                              SUBC1990
      K=K+1                                                             SUBC2000
      RK=K                                                              SUBC2010
      DO 10 KA=1,KEXTRA                                                 SUBC2020
      KB=KEXTRA-KA+1                                                    SUBC2030
10    STK(KB+1)=STK(KB)                                                 SUBC2040
      STK(1)=TOTAL                                                      SUBC2050
      ORDER=M+2*K+1                                                     SUBC2060
C     GENERATE THE NEXT ORDER OF LAGUERRE POLYNOMIAL FOR NEW K:         SUBC2070
      PM=P                                                              SUBC2080
      P=PP                                                              SUBC2090
      PP=P*(ORDER-ARG)-PM*SQRT(RK*(RM+RK))                              SUBC2100
      PP=PP/SQRT((RK+1.)*(RM+RK+1.))                                    SUBC2110
C     SET K TIMER TO PROVIDE EXTRA K TERMS AFTER CHECK < EPS:           SUBC2120
      KTIMER=KTIMER+1                                                   SUBC2130
      IF (K.GE.KLIMIT) GO TO 6                                          SUBC2140
      IF (CHECK.GE.EPS) KTIMER=0                                        SUBC2150
      IF (KTIMER.LE.KEXTRA) GO TO 3                                     SUBC2160
      GO TO 7                                                           SUBC2170
6     KOUT=KOUT+1                                                       SUBC2180
      IF (KEXTRA.EQ.0) GO TO 7                                          SUBC2190
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      TOTAL=C.                                                          SUBC2200
      DO 11 KA=1,KEXTRA                                                 SUBC2210
11    TOTAL=TOTAL+STK(KA+1)                                             SUBC2220
      RKX=KEXTRA                                                        SUBC2230
      TOTAL=TOTAL/RKX                                                   SUBC2240
C   END OF K LOOP;  ADVANCE M:                                          SUBC2250
7     M=M+1                                                             SUBC2260
      RM=M                                                              SUBC2270
      STP=SMS                                                           SUBC2280
      SMS=SMS*CMI+CMS*SMI                                               SUBC2290
      CMS=CMS*CMI-STP*SMI                                               SUBC2300
      IF (K.GT.KMAX) KMAX=K                                             SUBC2310
      FM=FM*RM                                                          SUBC2320
      DO 12 MA=1,MEXTRA                                                 SUBC2330
      MB=MEXTRA-MA+1                                                    SUBC2340
12    STM(MB+1)=STM(MB)                                                 SUBC2350
      STM(2)=TOTAL                                                      SUBC2360
C   SET M TIMER FOR EXTRA M TERMS:                                      SUBC2370
      IF (MS.EQ.1) MTIMER=MTIMER+1                                      SUBC2380
      IF (JSYM.GT.JMAX) GO TO 9                                         SUBC2390
      IF (K.GT.KEXTRA) MTIMER=0                                         SUBC2400
      IF (M.GE.MLIMIT) GO TO 13                                         SUBC2410
      IF (MTIMER.LE.MEXTRA) GO TO 2                                     SUBC2420
13    IF (MEXTRA.EQ.0) GO TO 9                                          SUBC2430
      TOTAL=C.                                                          SUBC2440
      DO 14 MA=1,MEXTRA                                                 SUBC2450
14    TOTAL=TOTAL+STM(MA+1)                                             SUBC2460
      RMX=MEXTRA                                                        SUBC2470
      TOTAL=TOTAL/RMX                                                   SUBC2480
C   END OF M LOOP;  COMPUTE OUTPUT SOLN.                                SUBC2490
9     MOLT=M-1                                                          SUBC2500
      IF (KOUT.EQ.0) KOUT=KMAX-1                                        SUBC2510
      SOLN=TOTAL*EXPON*APP/2.                                           SUBC2520
80    FORMAT ('   M=',I4,',  K=',I4,',  SUBTOTAL=',1PE10.3)            SUBC2530
88    FORMAT (2X,10E10.3)                                               SUBC2540
      RETURN                                                            SUBC2550
      END                                                               SUBC2560
C0000003                                                                SUBC2570
C                                                                       SUBC2580

      SUBROUTINE BD (M,K,G,H,SCF,B,D,JJMX6)                             SUBC2590
C                                                                       SUBC2600
C                                                                       SUBC2610
C   BD EVALUATES THE FIRST (B) AND SECOND (D) COEFFICIENTS IN THE       SUBC2620
C   INVERSION EQUATION, FOR A PARTICULAR SET OF INDEXES M & K.          SUBC2630
C   BD MAKES USE OF THE HERMITE POLYNOMIAL ARRAY GENERATED BY           SUBC2640
C   SUBROUTINE FIELD AS M & K ADVANCE.                                  SUBC2650
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C                                                                       SUB02660
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z   SUB02670
      COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM                      SUB02680
      DIMENSION G(IJMX),SCF(JJMX6),H(IIMX)                              SUB02690
      PIE=3.14159265358979                                              SUB02700
      B=0.                                                              SUB02710
      C=0.                                                              SUB02720
      RM=M                                                              SUB02730
      RK=K                                                              SUB02740
      RJMAX=JMAX                                                        SUB02750
      JJMX4=4*JJMX                                                      SUB02760
      DXI=2.*PIE/FCU                                                    SUB02770
      IF (JSYM.LE.0) GO TO 4                                            SUB02780
      IF (M.NE.0) GO TO 2                                               SUB02790
      S=DXI                                                             SUB02800
      DO 1 J=1,JMAX                                                     SUB02810
      DO 1 I=1,IMAX                                                     SUB02820
      II=I+1                                                            SUB02830
      IJ=IMAX*(J-1)+I                                                   SUB02840
      DH=H(II)-H(I)                                                     SUB02850
1     B=B+G(IJ)*S*DH                                                    SUB02860
      B=B*QSYM/2.                                                       SUB02870
      RETURN                                                            SUB02880
2     DO 3 J=1,JMAX                                                     SUB02890
      JS=J+JJMX4                                                        SUB02900
      S=SCF(JS)/RM                                                      SUB02910
      DO 3 I=1,IMAX                                                     SUB02920
      II=I+1                                                            SUB02930
      IJ=IMAX*(J-1)+I                                                   SUB02940
      DH=H(II)-H(I)                                                     SUB02950
3     B=B+G(IJ)*S*DH                                                    SUB02960
      B=B*QSYM                                                          SUB02970
      RETURN                                                            SUB02980
4     IF (M.NE.0) GO TO 6                                               SUB02990
      S=DXI                                                             SUB03000
      DO 5 J=1,JMAX                                                     SUB03010
      DO 5 I=1,IMAX                                                     SUB03020
      II=I+1                                                            SUB03030
      IJ=IMAX*(J-1)+I                                                   SUB03040
      DH=H(II)-H(I)                                                     SUB03050
5     B=B+G(IJ)*S*DH                                                    SUB03060
      B=B/2.                                                            SUB03070
      RETURN                                                            SUB03080
6     DO 7 J=1,JMAX                                                     SUB03090
      JS=J+JJMX4                                                        SUB03100
      J2=JS+JJMX                                                        SUB03110
      S=SCF(JS)/RM                                                      SUB03120
      C=SCF(J2)/RM                                                      SUB03130
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      DO 7 I=1,IMAX                                                     SUB03140
      II=I+1                                                            SUB03150
      IJ=IMAX*(J-1)+I                                                   SUB03160
      DH=H(II)-H(I)                                                     SUB03170
      B=B+G(IJ)*S*DH                                                    SUB03180
66    FORMAT ('  BD:',4I5,10F6.2)                                       SUB03190
7     D=D-G(IJ)*C*DH                                                    SUB03200
      RETURN                                                            SUB03210
      END                                                               SUB03220
CCOCC04                                                                 SUB03230
C                                                                       SUB03240

      SUBROUTINE FIELD2 (RS,SIG,SCLN,G,H,SCF,DGN)                       SUB03250
C                                                                       SUB03260
C   FIELD2 COMPUTES THE SAME INVERSION AS SUBROUTINE FIELD, EXCEPT THAT SUB03270
C   THE COEFFICIENTS B AND D ARE COMPUTED INDIVIDUALLY AS USED BY       SUB03280
C   CALLING BD.  DISK STORAGE IS NOT REQUIRED, BUT COMPUTING TIME IS    SUB03290
C   MUCH GREATER.  FIELD2 IS UTILIZED BY SPECIFYING A NEGATIVE MODE ON  SUB03300
C   THE INPUT PARAMETER.                                                SUB03310
C   FIELD EVALUATES THE VALUE OF THE FIELD FUNCTION AT A PARTICULAR     SUB03320
C   POINT DESIGNATED IN CYLINDRICAL COORDINATES, BY USING THE INVERSION SUB03330
C   EQUATION OF MALDONADO, ET AL.  FIELD CALLS SUBROUTINES BD & GAPRAY. SUB03340
C                                                                       SUB03350
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z   SUB03360
      COMMON /TAB/ INDEX,KEXTRA,MEXTRA,KLIMIT,MLIMIT,KOUT,MOUT         SUB03370
      COMMON /SYM/ ISYM,JSYM,MSYM,ECL,IMS,JMS,QSYM                     SUB03380
      DIMENSION G(IJMX),H(IIMX,5),SCF(JJMX,6)                           SUB03390
C     INITIALIZE THE VALUES:                                            SUB03400
      INDEX=0                                                           SUB03410
      MTIMER=0                                                          SUB03420
      KOUT=0                                                            SUB03430
      MOUT=0                                                            SUB03440
      MMAX=0                                                            SUB03450
      KMAX=0                                                            SUB03460
      TOTAL=0.                                                          SUB03470
      JJMX4=JJMX*6                                                      SUB03480
      IIMX2=(IIMX+1)/2                                                  SUB03490
      AR=ALPHA*RS                                                       SUB03500
      ARG=AR**2                                                         SUB03510
      EXPON=EXP(-ARG)                                                   SUB03520
      PIE=3.14159265358979                                              SUB03530
      APP=ALPHA/PIE/PIE                                                 SUB03540
      M=0                                                               SUB03550
      RM=M                                                              SUB03560
      RIMAX=IMAX                                                        SUB03570
      DX=2./RIMAX                                                       SUB03580
      RJMAX=JMAX                                                        SUB03590
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      DXI=2.*PIE/FCU                                                    SUPC3400
C     INITIALIZE THE MODIFIED HERMITE POLYNOMIAL ARRAY:  VECTORS:       SUPC3410
C     (1)=H1, (2)=H2, (3)=ALPHA*X(I), (4)=HM+2 STORED, (5)=HM+1 STORED  SUPC3420
      DO 1 II=1,IIMX2                                                   SUPC3430
      RII=II                                                            SUPC3440
      IIM=IIMX-II+1                                                     SUPC3450
      H(II,3)=ALPHA*(RII*DX-DX-1.)                                      SUPC3460
      H(IIM,3)=-H(II,3)                                                 SUPC3470
      H(II,1)=2.*H(II,3)                                                SUPC3480
      H(II,2)=(H(II,3)*H(II,1)-1.)/3.                                   SUPC3490
      H(IIM,1)=-H(II,1)                                                 SUPC3500
      H(IIM,2)=H(II,2)                                                  SUPC3510
      H(II,5)=H(II,2)                                                   SUPC3520
      H(IIM,5)=H(IIM,2)                                                 SUPC3530
      H(II,4)=H(II,1)                                                   SUPC3540
1     H(IIM,4)=H(IIM,1)                                                 SUPC3550
      SIGN=1.                                                           SUPC3560
      FM=1.                                                             SUPC3570
C     INITIATE THE SIN/COS ARRAY:                                       SUPC3580
      DO 11 J=1,JJMX                                                    SUPC3590
      RJM=J-1                                                           SUPC3600
      SCF(J,1)=0.                                                       SUPC3610
      SCF(J,2)=1.                                                       SUPC3620
      SCF(J,3)=SIN(RJM*DXI-PIE)                                         SUPC3630
      SCF(J,4)=COS(RJM*DXI-PIE)                                         SUPC3640
      SCF(J,5)=0.                                                       SUPC3650
11    SCF(J,6)=0.                                                       SUPC3660
      MS=0                                                              SUPC3670
C     COMMENCE THE M LOOP:                                              SUPC3680
2     SIGN=-SIGN                                                        SUPC3690
      K=0                                                               SUPC3700
      RK=K                                                              SUPC3710
      ARM=1.                                                            SUPC3720
      IF (M.NE.0) ARM=AR**M                                             SUPC3730
      KTIMER=0                                                          SUPC3740
      SIGNK=-1.                                                         SUPC3750
C     COMPUTE THE K=0 & K=1 ORDERS OF LAGUERRE POLYNOMIAL FOR GIVEN M:  SUPC3760
      PM=0.                                                             SUPC3770
      P=SQRT(1./FM)                                                     SUPC3780
      PP=(RM+1.-ARG)*SQRT(1./FM/(RM+1.))                                SUPC3790
C     ADVANCE THE SIN/COS ARRAY FOR NEW M:                              SUPC3800
      DO 12 J=1,JJMX                                                    SUPC3810
      SCF(J,1)=SCF(J,1)*SCF(J,4)+SCF(J,2)*SCF(J,3)                      SUPC3820
12    SCF(J,2)=SCF(J,2)*SCF(J,4)-SCF(J,1)*SCF(J,3)                      SUPC3830
      DO 13 J=1,JMAX                                                    SUPC3840
      SCF(J,5)=SCF(J+1,1)-SCF(J,1)                                      SUPC3850
13    SCF(J,6)=SCF(J+1,2)-SCF(J,2)                                      SUPC3860
C     TEST FOR SYMMETRY SKIPS:                                          SUPC3870
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      IF (MS.EQ.MSYM) MS=0                                                  SUBC4080
      TOTAL=C.                                                              SUBC4090
      MS=MS+1                                                               SUBC4100
      IF (MS.NE.1) GO TO 7                                                  SUBC4110
      RMS=RM*SIG                                                            SUBC4120
      CMS=COS(RMS)                                                          SUBC4130
      SMS=SIN(RMS)                                                          SUBC4140
C     COMMENCE THE K LOOP:                                                  SUBC4150
3     INDEX=INDEX+1                                                         SUBC4160
      SIGNK=-SIGNK                                                          SUBC4170
C     CALL THE B & D COEFFICIENTS AND COMPUTE THE M,K SUMMATION TERM:       SUBC4180
      CALL BD (M,K,G,H,SCF,B,D,JJMX6)                                       SUBC4190
      IF (DGN.LE.-2.) WRITE (6,89) M,K,B,D                                  SUBC4200
      BRAKET=B                                                              SUBC4210
      IF(RM.EQ.C.) GO TO 4                                                  SUBC4220
      BRAKET=B*CMS+D*SMS                                                    SUBC4230
4     ADD=SIGNK*BRAKET*P*ARM                                                SUBC4240
      TOTAL=TOTAL+ADD                                                       SUBC4250
C     ESTABLISH CHECK AS THE RELATIVE SIZE OF THE M,K TERM OF THE SERIES:   SUBC4260
      CHECK=ABS(ADD)                                                        SUBC4270
      IF (TOTAL.GT.EPS) CHECK=ABS(ADD/TOTAL)                                SUBC4280
C     ADVANCE THE K INDEX:                                                  SUBC4290
      K=K+1                                                                 SUBC4300
      RK=K                                                                  SUBC4310
      ORDER=M+2*K+1                                                         SUBC4320
C     GENERATE THE NEXT ORDER OF LAGUERRE POLYNOMIAL FOR NEW K:             SUBC4330
      PM=P                                                                  SUBC4340
      P=PP                                                                  SUBC4350
      PP=P*(ORDER-ARG)-PM*SQRT(RK*(RM+RK))                                  SUBC4360
      PP=PP/SQRT((RK+1.)*(RM+RK+1.))                                        SUBC4370
C     GENERATE THE NEXT ORDER OF THE SET OF HERMITE POLYNOMIALS FOR NEW K:  SUBC4380
      HA=SQRT(RK*(RK+RM))/ORDER                                             SUBC4390
      HB=2.*SQRT((RK+1.)*(RM+RK+1.))/(ORDER+1.)/(ORDER+2.)                  SUBC4400
      DO 5 II=1,IIMX2                                                       SUBC4410
      IIM=IIMX-II+1                                                         SUBC4420
      H(II,1)=2.*(H(II,3)*H(II,2)-HA*H(II,1))                               SUBC4430
      H(IIM,1)=SIGN*H(II,1)                                                 SUBC4440
5     H(II,2)=HB*(H(II,3)*H(II,1)-ORDER*H(II,2))                            SUBC4450
C     SET K TIMER TO PROVIDE EXTRA K TERMS AFTER CHECK < EPS:               SUBC4460
      KTIMER=KTIMER+1                                                       SUBC4470
      IF (K.GE.KLIMIT) GO TO 6                                              SUBC4480
      IF (CHECK.GE.EPS) KTIMER=0                                            SUBC4490
      IF (KTIMER.LE.KEXTRA) GO TO 3                                         SUBC4500
      GO TO 7                                                               SUBC4510
C     END OF K LOOP:  ADVANCE M AND COMPUTE NEW TOTAL:                      SUBC4520
6     KOUT=KOUT+1                                                           SUBC4530
7     M=M+1                                                                 SUBC4540
      IF (K.GT.KMAX) KMAX=K                                                 SUBC4550
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```
      RM=M                                                                  SUBC4560
      FM=FM*RM                                                              SUBC4570
C     REGENERATE THE HERMITE ARRAY FOR NEW M, K=0:                          SUBC4580
      DO 9 II=1,IIMX2                                                       SUBC4590
      IIM=IIMX-II+1                                                         SUBC4600
      H(II,2)=H(II,4)*SQRT(RM)/(RM+1.)                                      SUBC4610
      H(II,1)=H(II,5)*(RM+2.)                                               SUBC4620
      H(IIM,1)=-SIGN*H(II,1)                                                SUBC4630
      H(II,2)=2.*SQRT(RM+1.)*(H(II,3)*H(II,1)-(RM+1.)*H(II,2))              SUBC4640
      H(II,2)=H(II,2)/(RM+2.)/(RM+3.)                                       SUBC4650
      H(II,4)=H(II,1)                                                       SUBC4660
9     H(II,5)=H(II,2)                                                       SUBC4670
C     SET M TIMER FOR EXTRA M TERMS:                                        SUBC4680
      IF (MS.EQ.1) MTIMER=MTIMER+1                                          SUBC4690
      IF (JSYM.GT.JMAX) GO TO 9                                             SUBC4700
      IF (K.GT.KEXTRA) MTIMER=0                                             SUBC4710
      IF (M.GE.MLIMIT) GO TO 9                                              SUBC4720
      IF (MTIMER.LE.MEXTRA) GO TO 2                                         SUBC4730
C     END OF M LOOP:  COMPUTE OUTPUT SOLN.                                  SUBC4740
9     MOUT=M-1                                                              SUBC4750
      IF (KOUT.EQ.0) KOUT=KMAX-1                                            SUBC4760
      SOLN=TOTAL*EXPON*APP/2.                                               SUBC4770
99    FORMAT ('   M=',I4,',   K=',I4,',    R=',E10.4,',    D=',F10.4)       SUBC4780
      RETURN                                                                SUBC4790
      END                                                                   SUBC4800
                                                                            SUBC4810
                                                                            SUBC4820
CCCCCCC                                                                     SUBCC010
C                                                                           SUBCC020

      SUBROUTINE GARRAY (G,GA,NOF,DGN,NUMB,XO,YO,PHISYM)                    SUBCC030
C                                                                           SUBCC040
C     GARRAY FILLS THE DATA ARRAY OVER AN ORTHOGONAL AREA WITH              SUBCC050
C     THE REGULAR DATA OBTAINED BY THE METHOD CORRESPONDING TO THE          SUBCC060
C     PARTICULAR MODE:                                                      SUBCC070
C                                                                           SUBCC080
C          MODE 1  -  DATA OBTAINED BY SAMPLING A KNOWN FUNCTION SUPPLIED   SUBCC090
C                     IN SUBROUTINE FUNCT AND SAMPLED IN SUBROUTINE GOLF.   SUBCC100
C                                                                           SUBCC110
C          MODE 2  -  DATA OBTAINED BY GENERATING A REGULAR ARRAY FROM      SUBCC120
C                     IRREGULAR EXPERIMENTAL INPUT DATA READ IN.  CALLS     SUBCC130
C                     SUBROUTINE SHEET. (EXPERIMENTAL DATA MAY              SUBCC140
C                     BE SIMULATED, SEE 'SHEET')                            SUBCC150
C                                                                           SUBCC160
C          MODE 3  -  UTILIZES RAW DATA TAKEN AT THE PROPER INTERVAL,       SUBCC170
C                     OR PREVIOUSLY GENERATED, AND READ DIRECTLY INTO THE   SUBCC180
C                     GARRAY.  CALLS SUBROUTINE READ.                       SUBCC190
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C                                                                       SUBOC200
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z   SUBOC210
      COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM                      SUBOC220
      COMMON /IO/ CMS,IN1,IN2,IN4                                       SUBOC230
      DIMENSION G(IMAX,JMAX),GA(IMAX,JMAX)                              SUBOC240
      PIE=3.14159265358979                                              SUBOC250
      HS=SIZE/2.                                                        SUBOC260
      IF (MODE.GT.3) MODE=1                                             SUBOC270
      RIMX=IMAX                                                         SUBOC280
      RJMX=JMAX                                                         SUBOC290
      DELR=SIZE/RIMX                                                    SUBOC300
      DELXI=2.*PIE/FCU                                                  SUBOC310
      IF (MODE.GT.1) GO TO 2                                            SUBOC320
      DO 1 J=1,JMS                                                      SUBOC330
      RJ=J                                                              SUBOC340
      XI=(RJ-.5)*DELXI-PIE                                              SUBOC350
      J2=J+2*(JMS-J)                                                    SUBOC360
      J3=J+JMAX/2                                                       SUBOC370
      J4=J2+JMAX/2                                                      SUBOC380
      DO 1 I=1,IMS                                                      SUBOC390
      RI=I                                                              SUBOC400
      II=IMAX+1-I                                                       SUBOC410
      R=(RI-.5)*DELR-HS                                                 SUBOC420
      CALL GCLR (R,XI,GIJ,NCF,DGN,NUMB)                                 SUBOC430
      G(I,J)=GIJ                                                        SUBOC440
      IF (ISYM.EQ.2) G(II,J)=GIJ                                        SUBOC450
      IF (ISYM.EQ.2) GO TO 1                                            SUBOC460
      G(II,J3)=GIJ                                                      SUBOC470
      IF (JSYM.EQ.0) GO TO 1                                            SUBOC480
      G(I,J2)=GIJ                                                       SUBOC490
      G(II,J4)=GIJ                                                      SUBOC500
1     CONTINUE                                                          SUBOC510
      GO TO 4                                                           SUBOC520
2     IF (MODE.GT.2) GO TO 3                                            SUBOC530
      CALL SHEET (G,GA,XO,YO,PHISYM,NCF)                                SUBOC540
      GO TO 4                                                           SUBOC550
3     CALL READ (Z,XO,YO,PHISYM,NCF,IMAX,JMAX,G)                        SUBOC560
4     IF (DGN.GE.2) WRITE (6,39)                                        SUBOC570
      RETURN                                                            SUBOC580
39    FORMAT ('   GAPRAY RETURNS')                                      SUBOC590
      END                                                               SUBOC600
CCCCCCC                                                                 SUBOC610
C                                                                       SUBOC620
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      SUBROUTINE GOLF (R,XI,GIJ,NCF,DGN,NUMB)                          SUBC0630
C                                                                      SUBC0640
CCCC  GOLF COMPUTES THE FUNCTION G(R,XI) FOR A PARTICULAR LINE OF SIGHT SUBC0650
CCCC  FROM A KNOWN FUNCTION CONTAINED IN SUBROUTINE FUNCT.             SUBC0660
C                                                                      SUBC0670
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z  SUBC0680
      ZERO=0.                                                          SUBC0690
      LMAX=IMAX*3                                                      SUBC0700
      RLMAX=LMAX                                                       SUBC0710
      DELXP=SIZE/RLMAX                                                 SUBC0720
      SXI=SIN(XI)                                                      SUBC0730
      CXI=COS(XI)                                                      SUBC0740
      DELXS=DELXP*CXI                                                  SUBC0750
      DELYS=DELXP*SXI                                                  SUBC0760
      XP=DELXP*.5-SIZE/2.                                              SUBC0770
      XS=XP*CXI-R*SXI                                                  SUBC0780
      YS=XP*SXI+R*CXI                                                  SUBC0790
      GIJ=0.                                                           SUBC0800
      DO 1 L=1,LMAX                                                    SUBC0810
      RL=L                                                             SUBC0820
      CALL FUNCT(XS,YS,F,NCF,DGN,NUMB)                                 SUBC0830
      GIJ=GIJ+F                                                        SUBC0840
      XS=XS+DELXS                                                      SUBC0850
1     YS=YS+DELYS                                                      SUBC0860
      IF (GIJ.NE.0.) GIJ=GIJ*DELXP*BOX                                 SUBC0870
      IF ((SD.EQ.0.).OR.(NUMB.EQ.1)) GO TO 2                           SUBC0880
      IF (DGN.GE.3) WRITE (6,28) IX                                    SUBC0890
      CALL GAUSS (IX,SD,ZERO,RV)                                       SUBC0900
      GIJ=GIJ+RV                                                       SUBC0910
2     IF (DGN.GE.3) WRITE (6,29) R,XI,GIJ                              SUBC0920
      RETURN                                                           SUBC0930
29    FORMAT ('   R=',F8.3,',   XI=',F8.3,',   GIJ=',F8.3)            SUBC0940
28    FORMAT ('   GAUSS, IX=',I8)                                      SUBC0950
      END                                                              SUBC0960
CCOCCC7                                                                SUBC0970
C                                                                      SUBC0980


      SUBROUTINE FUNCT (XS,YS,F,NCF,DGN,NUMB)                          SUBC0990
C                                                                      SUBC1000
CCCCC                                                                  SUBC1010
CCCCC FUNCT EVALUATES AN INPUT FUNCTION AT POSITION (X,Y) IN THE TEST  SUBC1020
CCCCC SECTION COORDINATE SYSTEM.  NCF IDENTIFIES THE EQUATION USED.    SUBC1030
C                                                                      SUBC1040
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z  SUBC1050
      COMMON /EQPARA/ A,B,C,D,E,P,Q,S,T,U,V,W,RO,PA,NC,NA,N1,N2        SUBC1060
      DIMENSION RO(101),RA(101)                                        SUBC1070
      AA=A                                                             SUBC1080
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      BB=B                                                              SUBC1090
      CC=C                                                              SUBC1100
      DD=D                                                              SUBC1110
      EE=E                                                              SUBC1120
      PP=P                                                              SUBC1130
      IF (NUMP.LE.1) GO TO 50                                           SUBC1140
      AA=S                                                              SUBC1150
      BB=T                                                              SUBC1160
      CC=U                                                              SUBC1170
      DD=V                                                              SUBC1180
      EE=W                                                              SUBC1190
      PP=0                                                              SUBC1200
50    PIE=3.14159265358979                                              SUBC1210
      HS=SIZE/2.                                                        SUBC1220
      R=SQRT(XS**2+YS**2)/HS                                            SUBC1230
      F=0.                                                              SUBC1240
      IF (R.GT.1.) GO TO 11                                             SUBC1250
      IF (NOF.LE.0) GO TO 11                                            SUBC1260
C                                                                       SUBC1270
C     1.  AXISYMMETRIC GAUSSIAN:                                        SUBC1280
1     IF (NOF.GT.1) GO TO 2                                             SUBC1290
      F=AA*EXP(-1.*(R*HS/BB)**2)                                        SUBC1300
      GO TO 11                                                          SUBC1310
C                                                                       SUBC1320
C     2.  ADJUSTABLE RECTANGULAR STEP FUNCTION:                         SUBC1330
2     IF (NOF.GT.2) GO TO 3                                             SUBC1340
      F=PP                                                              SUBC1350
      IF ((ABS(XS-DD).LE.BB).AND.(ABS(YS-EE).LE.CC)) F=AA               SUBC1360
      GO TO 11                                                          SUBC1370
C                                                                       SUBC1380
C     3.  DISPLACABLE ELLIPTICAL GAUSSIAN:                              SUBC1390
3     IF (NOF.GT.3) GO TO 4                                             SUBC1400
      F=AA*EXP(-1.*(((XS-DD)/BB)**2+((YS-EE)/CC)**2))                   SUBC1410
      GO TO 11                                                          SUBC1420
C                                                                       SUBC1430
C     4.  CONSTANT:                                                     SUBC1440
4     IF (NOF.GT.4) GO TO 5                                             SUBC1450
      F=AA                                                              SUBC1460
      GO TO 11                                                          SUBC1470
C                                                                       SUBC1480
C     5.  ADJUSTABLE AND DISPLACABLE ELLIPTIC RAMP FUNCTION:            SUBC1490
5     IF (NOF.GT.5) GO TO 6                                             SUBC1500
      RBC=SQRT(((XS-DD)/BB)**2+((YS-EE)/CC)**2)                         SUBC1510
      F=0.                                                              SUBC1520
      IF (RBC.LT.1.) F=AA*((1.-RBC)**PP)                                SUBC1530
      GO TO 11                                                          SUBC1540
C                                                                       SUBC1550
C     6.  DISPLACABLE ELLIPTIC STEP FUNCTION:                           SUBC1560
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6       IF (NCF.GT.6) GO TO 7                                                   SUBC157C
        RBC=SQRT(((XS-DD)/BB)**2+((YS-EE)/CC)**2)                               SUBC158C
        F=0.                                                                    SUBC159C
        IF (RBC.LT.1.) F=AA                                                     SUBC160C
        GO TO 11                                                                SUBC161C
C                                                                               SUBC162C
C     7.  CIRCULAR COSINE-SQUARED FUNCTION OF BB MAXIMA:                        SUBC163C
7       IF (NCF.GT.7) GO TO 8                                                   SUBC164C
        F=AA*COS((2.*BB-1.)*PIE*R/2.)**2                                        SUBC165C
        GO TO 11                                                                SUBC166C
C                                                                               SUBC167C
C     8.  NUMERICAL FUNCTION:  REQUIRES AN INPUT ARRAY READ IN BY               SUBC168C
C         SUBROUTINE FREAD: N FOLLOWED BY N POINT VALUES. (101 MAX)             SUBC169C
C         A CONSTANT VALUE AA IS ADDED TO THE FUNCTION.                         SUBC170C
8       IF (NCF.GT.8) GO TO 9                                                   SUBC171C
        IF (NUMB.LE.1) N=NC                                                     SUBC172C
        IF (NUMB.GT.1) N=NA                                                     SUBC173C
        NM=N-1                                                                  SUBC174C
        NMM=N-2                                                                 SUBC175C
        RN=N                                                                    SUBC176C
        RI=R*(RN-1.)+1.                                                         SUBC177C
        IR=INT(RI)                                                              SUBC178C
        RIR=FLOAT(IR)                                                           SUBC179C
        DI=RI-RIR                                                               SUBC180C
        IF (NUMB.LE.1) F=RO(IR)                                                 SUBC181C
        IF (NUMB.GT.1) F=RA(IR)                                                 SUBC182C
        IF ((IR.NE.N).AND.(NUMB.LE.1)) F=F+DI*(RO(IR+1)-RO(IR))                 SUBC183C
        IF ((IR.NE.N).AND.(NUMB.GT.1)) F=F+DI*(RA(IR+1)-RA(IR))                 SUBC184C
        F=F*AA+BB                                                               SUBC185C
        GO TO 11                                                                SUBC186C
C                                                                               SUBC187C
C     9.  SPECIAL FUNCTION:  MAY BE WRITTEN FOR THE OCCASION AND                SUBC188C
C         INSERTED IN SUBROUTINE SPFUN                                          SUBC189C
9       IF (NCF.GT.9) GO TO 10                                                  SUBC190C
        CALL SPFUN (XS,YS,F)                                                    SUBC191C
        GO TO 11                                                                SUBC192C
C                                                                               SUBC193C
C     EQUATIONS NO. 10 AND BEYOND ARE SET TO ZERO.                              SUBC194C
10      F=C.                                                                    SUBC195C
C                                                                               SUBC196C
C                                                                               SUBC197C
11      IF (DCN.GE.4) WRITE (6,99) XS,YS,F                                      SUBC198C
99      FORMAT ('   XS=',F8.3,',   YS=',F8.3,',   F=',F8.3)                    SUBC199C
        RETURN                                                                  SUBC200C
        END                                                                     SUBC201C
CCCCCCC                                                                         SUBC202C
C                                                                               SUBC203C
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      SUBROUTINE SPFUN (XS,YS,F)                                        SUB02040
C                                                                       SUB02050
C     SPFUN IS A SPECIAL ROUTINE FOR EQ'N NO. 9.  ANY FUNCTION MAY BE   SUB02060
C     ENTERED.                                                          SUB02070
C                                                                       SUB02080
      COMMON /EQPARA/ A,B,C,D,E,P,Q,S,T,U,V,W,RO,RA,NC,NA,N1,N2         SUB02090
      DIMENSION RO(101),RA(101)                                         SUB02100
      F=C.                                                              SUB02110
      IF ((ABS(XS).LE.B).AND.(ABS(YS).LE.C))  F=A                       SUB02120
      RETURN                                                            SUB02130
      END                                                               SUB02140
CCCCCC                                                                  SUB02150
C                                                                       SUB02160


      SUBROUTINE SHEET (G,D,XC,YC,PHISYM,NCF)                           SUB02170
C                                                                       SUB02180
C     SHEET READS IRREGULARLY SPACED VALUES OF THE LINE INTEGRAL, AS    SUB02190
C     OBTAINED FROM HOLOGRAPHIC INTERFEROGRAMS.  THE INTEGRAL LINES MAY BESUB02200
C     DEFINED EITHER BY GRID INTERCEPT POSITIONS, OR BY ANGLE AND RADIUS SUB02210
C     ABOUT THE CENTER OF THE LABORATORY COORDINATE SYSTEM CENTER.  LINES SUB02220
C     MUST BE ENTERED IN CONSECUTIVE ORDER FROM LOWEST (NEG.) TO HIGHEST SUB02230
C     (POS.) RADIUS.  DATA MAY BE SIMULATED BY SPECIFYING NCODE=1,      SUB02240
C     FOLLOWED BY APERATURE POSITIONS FOR A FUNCTION NUMBER IN 'SUBFUNCT'.SUB02250
C                                                                       SUB02260
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z   SUB02270
      COMMON /SYM/ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM                       SUB02280
      COMMON /IC/ CMS,IN1,IN2,IN4                                       SUB02290
      DIMENSION G(IMAX,JMAX),D(IMAX,JMAX),XI(303),RR(303)               SUB02300
      DIMENSION XG(303),XD(303),YG(303),YD(303),XY(303)                 SUB02310
      NAR=303                                                           SUB02320
      PIE=3.14159265358979                                              SUB02330
      MPIE=-PIE                                                         SUB02340
      TPIE=2.*PIE                                                       SUB02350
      MPIT=MPIE/2.                                                      SUB02360
      PIHT=PIE/2.                                                       SUB02370
C     ZERO THE ARRAYS:                                                  SUB02380
      DO 1 J=1,JMAX                                                     SUB02390
      DO 1 I=1,IMAX                                                     SUB02400
      G(I,J)=C.                                                         SUB02410
1     D(I,J)=C.                                                         SUB02420
      DO 2 I=1,NAR                                                      SUB02430
      XG(I)=C.                                                          SUB02440
      XD(I)=C.                                                          SUB02450
      YG(I)=C.                                                          SUB02460
      YD(I)=C.                                                          SUB02470
      XY(I)=C.                                                          SUB02480
      XI(I)=C.                                                          SUB02490
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2       RR(I)=C.                                                                SUB02500
C     READ THE BASIC DATA:                                                      SUB02510
        IF (CMS.EQ.1.) REWIND 1                                                 SUB02520
        READ (IN1,59) NOF,NCODE                                                 SUB02530
        READ (IN1,58) Z,XO,YO,PHISYM,XMX,XMN,YMX,YMN                            SUB02540
        READ (IN1,59) JM                                                        SUB02550
        RIMX=IMAX                                                               SUB02560
        DR=SIZE/RIMX                                                            SUB02570
        R=(-DR-SIZE)/2.                                                         SUB02580
        RZO=SQRT(XO**2+YO**2)                                                   SUB02590
        GAM=ATANM(YO,XO)                                                        SUB02600
        TP=3-ISYM                                                               SUB02610
        BT=JSYM                                                                 SUB02620
        DAN=PIE*TP/BT                                                           SUB02630
        HS=SIZE/2.                                                              SUB02640
        MXY=1                                                                   SUB02650
        IF((XMX.NE.0.).OR.(XMN.NE.0.).OR.(YMX.NE.0.).OR.(YMN.NE.0.))MXY=0       SUB02660
        IF (MXY.EQ.1) GO TO 3                                                   SUB02670
        XMX=XO+HS                                                               SUB02680
        YMX=YO+HS                                                               SUB02690
        XMN=XO-HS                                                               SUB02700
        YMN=YO-HS                                                               SUB02710
C     COMMENCE THE READ AND FILL LOOP:                                          SUB02720
3       DO 12 J=1,JM                                                            SUB02730
        READ (IN1,59)IM                                                         SUB02740
        MN=0                                                                    SUB02750
        XH=0.                                                                   SUB02760
        YH=0.                                                                   SUB02770
C     READ THE LINES, DETERMINE CODE, CALCULATE RADIUS & ANGLE FOR CODE 1:      SUB02780
        DO 5 I=1,IM                                                             SUB02790
        IF(NCODE.LE.0)READ(IN1,58) XD(I),YD(I),XG(I),YG(I),D(I,J),RR(I),        SUB02800
       1XI(I),XY(I)                                                             SUB02810
        IF(NCODE.GE.1)CALL SIM(XD(I),YD(I),XG(I),YG(I),RR(I),XI(I),             SUB02820
       1XY(I),XO,YO,PHISYM,XMX,XMN,YMX,YMN,NOF,I,IM)                            SUB02830
        IF (XY(I).EQ.3.) GO TO 5                                                SUB02840
        IF((XD(I).NE.0.).OR.(YD(I).NE.0.)) XY(I)=1.                             SUB02850
        IF((XG(I).NE.0.).OR.(YG(I).NE.0.)) XY(I)=1.                             SUB02860
        IF(((RR(I).NE.0.).OR.(XI(I).NE.0.)).AND.(XY(I).EQ.0.)) XY(I)=2.         SUB02870
        IF((XY(I).EQ.0.).AND.(D(I,J).NE.0.)) XY(I)=2.                           SUB02880
        IF (XY(I).NE.1.) GO TO 4                                                SUB02890
        DEN=SQRT((XG(I)-XD(I))**2+(YG(I)-YD(I))**2)                             SUB02900
        IF (DEN.EQ.0.) XY(I)=4.                                                 SUB02910
        IF (XY(I).EQ.4.) GO TO 4                                                SUB02920
        RR(I)=((XO-XD(I))*(YG(I)-YD(I))-(XG(I)-XD(I))*(YO-YD(I)))/DEN           SUB02930
        XI(I)=ATANM((YG(I)-YD(I)),(XG(I)-XD(I)))                                SUB02940
        XIM=XI(I)                                                               SUB02950
4       IF (XY(I).EQ.2.) XIM=XI(I)                                              SUB02960
        XIN=XIM                                                                 SUB02970
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      IF(XY(I).EQ.2.) RR(I)=RR(I)+RZC*SIN(GAM-XI(I))                    SUBC2980
5     CONTINUE                                                          SUBC2990
C     COMPUTE MAX AND MIN ANGLE INDEXES FOR APERATURE POSITION LOCATION: SUBC3000
      DO 6 I=1,IM                                                       SUBC3010
      IF((XY(I).NE.1.).OR.(XY(I).NE.2.)) GO TO 6                        SUBC3020
      IF (XI(I).GT.XIM) XIM=XI(I)                                       SUBC3030
      IF (XI(I).GT.XIM) IMT=I                                           SUBC3040
      IF (XI(I).LT.XIN) XIN=XI(I)                                       SUBC3050
      IF (XI(I).LE.XIN) INT=I                                           SUBC3060
6     CONTINUE                                                          SUBC3070
C     DETERMINE APERATURE LOCATION:                                     SUBC3080
      LPR=0                                                             SUBC3090
      XIC=XI(IMT)-XI(INT)                                               SUBC3100
      IF(ABS(XIC).LT..0000) LPR=1                                       SUBC3110
      XIH=(XI(IMT)+XI(INT))/2.                                          SUBC3120
      RRH=10000.                                                        SUBC3130
      XH=RRH*COS(XIH)                                                   SUBC3140
      YH=RRH*SIN(XIH)                                                   SUBC3150
      IF (LPR.EQ.1) GO TO 7                                             SUBC3160
      YTX=-RR(IMT)*SIN(XI(IMT))-YC                                      SUBC3170
      YTN=-RR(INT)*SIN(XI(INT))-YC                                      SUBC3180
      XTX=RR(IMT)*COS(XI(IMT))-XC                                       SUBC3190
      XTN=RR(INT)*COS(XI(INT))-XC                                       SUBC3200
      UA=TAN(XI(IMT))                                                   SUBC3210
      UC=TAN(XI(INT))                                                   SUBC3220
      UB=YTX-UA*XTX                                                     SUBC3230
      UD=YTN-UC*XTN                                                     SUBC3240
      XH=(UD-UB)/(UA-UC)                                                SUBC3250
      YH=XH*UA+UB                                                       SUBC3260
      RRH=SQRT((XH-XC)**2+(YH-YC)**2)                                   SUBC3270
      XIH=ATANM((YH-YC),(XH-XC))                                        SUBC3280
7     CONTINUE                                                          SUBC3290
C     FILL THE ANGLE AND RADIUS FOR ANY CODE 3 OR 4 LINES:              SUBC3300
      DO 9 I=1,IM                                                       SUBC3310
      IF(XY(I).NE.3.) GO TO 8                                           SUBC3320
      BAS=SQRT(RRH**2-RR(I)**2)                                         SUBC3330
      XI(I)=XIH-ATANM(RR(I),BAS)                                        SUBC3340
      GO TO 9                                                           SUBC3350
8     XI(I)= ATANM((YH-YD(I)),(XH-XD(I)))                               SUBC3360
      RR(I)=RRH*SIN(XI(I)-XIH)                                          SUBC3370
9     CONTINUE                                                          SUBC3380
C     ANGLES AND RADII ARE NOW FILLED FOR ALL POINTS IN THIS LINE.      SUBC3390
C     VACATE THE SET OF VECTORS TO BE USED AS TEMPORARY STORAGE:        SUBC3400
      DO 10 I=1,IM                                                      SUBC3410
      XD(I)=0.                                                          SUBC3420
      YD(I)=0.                                                          SUBC3430
      XG(I)=0.                                                          SUBC3440
      YG(I)=RR(I)                                                       SUBC3450
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      XY(I)=D(I,J)                                                      SUPC34460
      RR(I)=C.                                                          SUPC34470
      D(I,J)=0.                                                         SUPC34480
10    XI(I)=C.                                                          SUPC34490
C   CONVERT THE LINE TO REGULAR RADII USING INTERPOLATION:              SUPC34500
      RR(1)=R+DR                                                        SUPC34510
      CALL SPLINE(YG,XY,IM,RR(1),D(1,J))                                SUPC34520
      DO 11 I=2,IMAX                                                    SUPC34530
      FI=I                                                              SUPC34540
      RR(I)=R+DR*FI                                                     SUPC34550
11    CALL SPLINN(YG,XY,IM,RR(I),D(I,J))                                SUPC34560
C   GENERATE THE VECTOR OF ANGLES FOR THIS COLUMN AND STORE IN G ARRAY: SUPC34570
      DO 12 I=1,IMAX                                                    SUPC34580
      BAS=SQRT(RRH**2-RR(I)**2)                                         SUPC34590
      G(I,J)=XIH-ATANM(RR(I),BAS)                                       SUPC34600
      YG(I)=C.                                                          SUPC34610
      D(I,J)=XY(I)                                                      SUPC34620
12    XY(I)=C.                                                          SUPC34630
C   COLUMNS ARE NOW ALL REGULARLY FILLED.                               SUPC34640
C   NEXT,INTERPOLATE EACH ROW REGULARLY OVER THE ANGLES.                SUPC34650
      DO 23 I=1,IMAX                                                    SUPC34660
C   EXPAND THE DATA TO 2 SETS TO ESTABLISH SMOOTH INTERPOLATION.        SUPC34670
      JM3=3*JM                                                          SUPC34680
      II=IMAX+1-I                                                       SUPC34690
      IF (JSYM.NE.0) GO TO 14                                           SUPC34700
      DO 13 J=1,JMS                                                     SUPC34710
      J2=J+JMS                                                          SUPC34720
      J3=J2+JMS                                                         SUPC34730
      XD(J)=D(I,J)                                                      SUPC34740
      XD(J2)=D(II,J)                                                    SUPC34750
      XD(J3)=D(I,J)                                                     SUPC34760
      XG(J)=G(I,J)-TPIE-PHISYM                                          SUPC34770
      XG(J2)=G(II,J)-PIE-PHISYM                                         SUPC34780
13    XG(J3)=G(I,J)-PHISYM                                              SUPC34790
      GO TO 16                                                          SUPC34800
14    DO 15 J=1,JMS                                                     SUPC34810
      J1=JMS+1-J                                                        SUPC34820
      J2=JMS+J                                                          SUPC34830
      J3=JM3+1-J                                                        SUPC34840
      XD(J1)=D(I,J)                                                     SUPC34850
      XD(J2)=D(II,J)                                                    SUPC34860
      XD(J3)=D(II,J)                                                    SUPC34870
      XG(J1)=G(I,J)-2.*(G(I,J)-PHISYM)-PIE-PHISYM                       SUPC34880
      XG(J2)=G(II,J)-PIE-PHISYM                                         SUPC34890
15    XG(J3)=G(II,J)+2.*(DAN+PHISYM-G(II,J))-PIE-PHISYM                 SUPC34900
16    CONTINUE                                                          SUPC34910
      JM2=2*JMS                                                         SUPC34920
      JP=JMS/2                                                          SUPC34930
```

```
      DO 17 J=1,JM2                                                     SUBC3940
      XD(J)=XD(J+JP)                                                    SUBC3950
17    XG(J)=XG(J+JP)                                                    SUBC3960
      JJS=JM2+1                                                         SUBC3970
      DO 18 J=JJS,JM3                                                   SUBC3980
      XD(J)=0.                                                          SUBC3990
18    XG(J)=0.                                                          SUBC4000
C   FIND THE SMALLEST ANGLE                                             SUBC4010
      XY(1)=1.                                                          SUBC4020
      SA=XG(1)                                                          SUBC4030
      DO 19 J=1,JM2                                                     SUBC4040
      IF (XG(J).GE.SA) GO TO 19                                         SUBC4050
      SA=XG(J)                                                          SUBC4060
      XY(1)=J                                                           SUBC4070
19    CONTINUE                                                          SUBC4080
C   FIND THE MAX ANGLE IN THE ROW:                                      SUBC4090
      XY(JM2)=JM2                                                       SUBC4100
      SB=XG(JM2)                                                        SUBC4110
      DO 20 J=1,JM2                                                     SUBC4120
      IF (XG(J).LE.SB) GO TO 20                                         SUBC4130
      SB=XG(J)                                                          SUBC4140
      XY(JM2)=J                                                         SUBC4150
20    CONTINUE                                                          SUBC4160
C   DETERMINE THE ORDER OF INCREASING ANGLE IN THE ROW                  SUBC4170
      SB=XG(JM2)                                                        SUBC4180
      JJ=2                                                              SUBC4190
21    JSA=XY(JJ-1)                                                      SUBC4200
      SA=XG(JJ-1)                                                       SUBC4210
      JTS=0                                                             SUBC4220
      DO 22 J=1,JM2                                                     SUBC4230
      IF (XG(J).LE.SA) GO TO 22                                         SUBC4240
      IF (XG(J).GT.SB) GO TO 22                                         SUBC4250
      SB=XG(J)                                                          SUBC4260
      XY(JJ)=J                                                          SUBC4270
      JTS=1                                                             SUBC4280
22    CONTINUE                                                          SUBC4290
      IF (JTS.EQ.0) JM2=JJ                                              SUBC4300
      JJ=JJ+1                                                           SUBC4310
      IF (JJ.LE.JM2) GO TO 21                                           SUBC4320
      DO 23 J=1,JM2                                                     SUBC4330
      JX=XY(J)                                                          SUBC4340
23    YD(J)=XD(JX)                                                      SUBC4350
C   INTERPOLATE:                                                        SUBC4360
      DXI=2.*PIE/FCU                                                    SUBC4370
      XI(J)=DXI/2.-PIE-PHISYM                                           SUBC4380
      CALL SPLINE (XG,YD,JM2,XI(J),G(I,J))                              SUBC4390
      DO 24 J=2,JMS                                                     SUBC4400
      XI(J)=XI(J-1)+DXI                                                 SUBC4410
```

```
24      CALL SPLINN (XG,YD,JM2,XI(J),G(I,J))                              SUB04420
        DO 25 J=1,JMS                                                     SUB04430
        XIJ=XI(J)                                                         SUB04440
        XU=XMX                                                            SUB04450
        IF ((XIJ.GE.0.).AND.(XIJ.LT.PIE)) XU=XMN                          SUB04460
        YU=YMN                                                            SUB04470
        IF ((XIJ.GE.MPIT).AND.(XIJ.LT.PIT)) YU=YMX                        SUB04480
        XL=XMN                                                            SUB04490
        IF ((XIJ.GE.0.).AND.(XIJ.LT.PIE)) XL=XMX                          SUB04500
        YL=YMX                                                            SUB04510
        IF ((XIJ.GE.MPIT).AND.(XIJ.LT.PIT)) YL=YMN                        SUB04520
        SXIJ=SIN(XIJ)                                                     SUB04530
        CXIJ=COS(XIJ)                                                     SUB04540
        RMN=(XC-XL)*SXIJ-(YC-YL)*CXIJ                                     SUB04550
        RMX=(XC-XL)*SXIJ-(YC-YU)*CXIJ                                     SUB04560
        DO 25 I=1,IMAX                                                    SUB04570
        IF (RR(I).LT.RMN) G(I,J)=0.                                       SUB04580
        IF (RR(I).GT.RMX) G(I,J)=0.                                       SUB04590
25      CONTINUE                                                          SUB04600
C     EXPAND SYMMETRY SECTOR INTO AN ORTHOGONAL INTERVAL.                 SUB04610
        IF (ISYM.EQ.2) GO TO 27                                           SUB04620
        DO 26 J=1,JMS                                                     SUB04630
        J2=JMAX/2+1-J                                                     SUB04640
        J3=JMAX/2+J                                                       SUB04650
        J4=JMAX+1-J                                                       SUB04660
        DO 26 I=1,IMAX                                                    SUB04670
        II=IMAX+1-I                                                       SUB04680
        G(I,J2)=G(I,J)                                                    SUB04690
        G(II,J3)=G(I,J)                                                   SUB04700
26      G(II,J4)=G(I,J)                                                   SUB04710
        RETURN                                                            SUB04720
C     FOR EVEN SYMMETRY, AVERAGE THE GARRAY COLUMNS.                      SUB04730
27      IMS=(2*IMAX+1)/2                                                  SUB04740
        DO 28 J=1,JMAX                                                    SUB04750
        DO 28 I=1,IMS                                                     SUB04760
        II=IMAX+1-I                                                       SUB04770
        GST=(G(I,J)+G(II,J))/2.                                           SUB04780
        G(I,J)=GST                                                        SUB04790
28      G(II,J)=GST                                                       SUB04800
        RETURN                                                            SUB04810
50      FORMAT (5I5)                                                      SUB04820
58      FORMAT (1CF7.3)                                                   SUB04830
        END                                                               SUB04840
CCCCC10                                                                   SUB04850
C                                                                         SUB04860
```

```
      FUNCTION ATANM(Y,X)                                                SUBC4870
C                                                                        SUBC4880
C    COMPUTES THE ARCTAN OF Y/X BETWEEN -PI AND +PI.                     SUBC4890
C                                                                        SUBC4900
      PIE=3.14159265358979                                               SUBC4910
      PI2=PIE/2.                                                         SUBC4920
      ATANM=SIGN(PI2,Y)                                                  SUBC4930
      IF(X.NE.0.) ATANM=ATAN(Y/X)                                        SUBC4940
      IF(X.GE.0.) RETURN                                                 SUBC4950
      IF(Y.GE.0.) ATANM=PIE+ATANM                                        SUBC4960
      IF(Y.LT.0.) ATANM=-PIE+ATANM                                       SUBC4970
      RETURN                                                             SUBC4980
      END                                                                SUBC4990
C0000011                                                                 SUBC5000
C                                                                        SUBC5010

      SUBROUTINE SIM (XD,YD,XG,YG,D,R,XI,XY,XC,YC,PS,XM,XN,YM,YN,I,IM,   SUBC5020
     1DG,NF)                                                             SUBC5030
C    SIM SIMULATES THE FRINGE NUMBER DATA ONE WOULD OBTAIN FROM THE      SUBC5040
C    HOLOGRAPHIC INTERFEROGRAM PROCESS FOR A KNOWN FUNCTION AS           SUBC5050
C    CONTAINED IN SUBROUTINE FUNCT.  THE GRID BOX DIMENSIONS MUST        SUBC5060
C    EXCEED THE INVERSION CIRCLE SIZE, AND APERATURE POINTS SPECIFIED    SUBC5070
C    MUST FALL BETWEEN XI=-40 DEGREES, AND XI=+130 DEGREES.              SUBC5080
C                                                                        SUBC5090
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALF,SIZ,EPS,MOD,BOX,SD,IX,Z        SUBC5100
      COMMON /IO/ CMS,IN1,IN2,IN4                                        SUBC5110
      READ (IN1,2S) XH,YH                                                SUBC5120
      ZER=0.                                                             SUBC5130
      RIM=IM                                                             SUBC5140
      RI=I                                                               SUBC5150
      DX=(XM-XN)/(RIM-1.)                                                SUBC5160
      DY=(YM-YN)/(RIM-1.)                                                SUBC5170
      X1=XM-(RIM-1.)*DX                                                  SUBC5180
      Y1=YM-(RIM-1.)*DY                                                  SUBC5190
      XI1=ATANM (YH-Y1,XH-X1)-PS                                         SUBC5200
      RRH=SQRT((XH-XD)**2+(YH-YD)**2)                                    SUBC5210
      XIC=ATANM(YH-YD,XH-XC)-PS                                          SUBC5220
      R1=RRH*SIN(XI1-XIC)                                                SUBC5230
      IF ((I.GT.1).AND.(I.LT.IM)) GO TO 1                                SUBC5240
      IF (ABS(R1).LT.SIZ/2.) R1=SIGN(SIZ/2.,R1)                          SUBC5250
      XY=3.                                                              SUBC5260
      GO TO 2                                                            SUBC5270
1     CALL GCLF (R,XI,D,NR,ZER,ZER)                                      SUBC5280
      XD=XN                                                              SUBC5290
      YD=YN                                                              SUBC5300
      IF (XH.NE.X1) YD=Y1-(X1-XN)*(YH-Y1)/(XH-X1)                        SUBC5310
      IF (YS.GE.YN) GO TO 2                                              SUBC5320
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      YD=YN                                                                  SUP05330
      IF (YH.NE.Y1) XD=X1-(Y1-YN)*(XH-X1)/(YH-Y1)                            SUP05340
2     RETURN                                                                 SUP05350
29    FORMAT (1CF7.3)                                                        SUP05360
      END                                                                    SUP05370
                                                                             SUP05380
CCCCC12                                                                      SUPCC010
C                                                                            SUPCCC20

      SUBROUTINE FREAD (NC,RC,NF,ZZ)                                         SUPCCC30
C                                                                            SUPCCC40
C   FREAD READS THE NUMERIC ARRAY WHICH IS USED FOR EQUATION 8 OF            SUPCCC50
C   SUBROUTINE FUNCT.  FIRST CARD IS NUMBER OF POINTS (N.GE.1),              SUPCCC60
C   FOLLOWED BY ONE POINT PER CARD.                                          SUPCCC70
C                                                                            SUPCCC80
      DIMENSION RC(101)                                                      SUPCCC90
      READ (NF,99) NC,ZZ                                                     SUPCC100
      READ (NF,98) (RC(I),I=1,NC)                                            SUPCC110
99    FORMAT (I5,F9.3)                                                       SUPCC120
98    FORMAT (F8.3)                                                          SUPCC130
      RETURN                                                                 SUPCC140
      END                                                                    SUPCC150
CCCCC13                                                                      SUPCC160
C                                                                            SUPCC170

      SUBROUTINE GPRINT (G,NUMB)                                             SUPCC180
C                                                                            SUPCC190
C   GPRINT PRINTS THE DATA ARRAY 'G'  WHICH WAS INPUT TO                     SUPCC200
C   THE PROGRAM IN SUBROUTINE GARRAY.                                        SUPCC210
C                                                                            SUPCC220
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z        SUPCC230
      DIMENSION G(IJMX)                                                      SUPCC240
      DIMENSION X(15)                                                        SUPCC250
      DATA HYP,VERT/1H-,1H|/                                                 SUPCC260
      IF (NUMB.EQ.1) WRITE (6,99) MODE,Z                                     SUPCC270
      IF (NUMB.EQ.2) WRITE (6,92) Z                                          SUPCC280
      JM2=JMAX/2                                                             SUPCC290
      RIMAX=IMAX                                                             SUPCC300
      RJMAX=JMAX                                                             SUPCC310
      DX=SIZE/RIMAX                                                          SUPCC320
      DXI=360./RJMAX                                                         SUPCC330
C   INTRVL SETS THE NUMBER OF TERMS PRINTED PER LINE. IF IT IS ALTERED,      SUPCC340
C   ONE MUST ALSO REDIMENSION X AND ALTER FORMATS 98, 97, AND 95.            SUPCC350
      INTRVL=15                                                              SUPCC360
      IR=1                                                                   SUPCC370
      IT=IR+INTRVL-1                                                         SUPCC380
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```
1       IF (IT.GT.IMAX) IT=IMAX                                                 SUPCC390
        IBT=IT-IB+1                                                             SUPCC400
        WRITE (6,98) (II,II=IB,IT)                                              SUPCC410
        DO 2 I=1,IBT                                                            SUPCC420
        RI=IB-1+I                                                               SUPCC430
2       X(I)=-SIZE/2.+(RI-.5)*DX                                                SUPCC440
        LM=7*IBT+1                                                              SUPCC450
        WRITE (6,97) (X(I),I=1,IBT)                                             SUPCC460
        WRITE (6,96) (HYP,L=1,LM),VERT                                          SUPCC470
        JMH=JM2+1                                                               SUPCC480
        DO 3 J=JMH,JMAX                                                         SUPCC490
        RJ=J                                                                    SUPCC500
        XI=-180.+DXI*(RJ-.5)                                                    SUPCC510
        IGB=(J-1)*IMAX+IB                                                       SUPCC520
        IGT=IGB-IB+IT                                                           SUPCC530
3       WRITE (6,95) J,XI,(G(L),L=IGB,IGT)                                      SUPCC540
        WRITE (6,94) (HYP,L=1,LM),VERT                                          SUPCC550
        IB=IB+INTRVL                                                            SUPCC560
        ITOLD=IT                                                                SUPCC570
        IT=IT+INTRVL                                                            SUPCC580
        IF (ITOLD.LT.IMAX) GO TO 1                                              SUPCC590
        WRITE (6,93)                                                            SUPCC600
99      FORMAT (1H1//'    THE ARRAY OF INPUT DATA (G), OBTAINED BY GARRAY,      SUPCC610
      1MODE ',I1,', FOR Z='F7.3,' CM:')                                         SUPCC620
98      FORMAT (///11X,'I= ',15I7)                                              SUPCC630
97      FORMAT (11X,'X=  ',15F7.3)                                              SUPCC640
96      FORMAT ('    J=      XI= |',115A1)                                      SUPCC650
95      FORMAT (2X,I3,F9.2,'|',15F7.3)                                          SUPCC660
94      FORMAT (14X,'|',115A1)                                                  SUPCC670
93      FORMAT (///5X)                                                          SUPCC680
92      FORMAT (1H1//'    THE ADD-ON FUNCTION GARRAY FOR Z=',F7.3,' CM:')       SUPCC690
        RETURN                                                                  SUPCC700
        END                                                                     SUPCC710
CCOOC14                                                                         SUPCC720
C                                                                               SUPCC730


        SUBROUTINE GPUNCH (Z,XC,YO,PHS,NCF,IMX,JMX,G)                           SUPCC740
C                                                                               SUPCC750
C    GPUNCH PUNCHES OUT THE FIRST NON-SYMMETRIC PORTION OF GARRAY               SUPCC760
C    (OR WRITES IT ON FILE 7 IN CMS VERSION)                                    SUPCC770
C                                                                               SUPCC780
        COMMON /SYM/ ISM,JSM,MSM,FCU,IMS,JMS,QSM                                SUPCC790
        DIMENSION G(IMX,JMX)                                                    SUPCC800
        WRITE (7,39) NCF,IMX,JMX,ISM,JSM,IMS,JMS                                SUPCC810
        WRITE (7,38) ((G(I,J),I=1,IMS),J=1,JMS)                                 SUPCC820
39      FORMAT(1CI5)                                                            SUPCC830
38      FORMAT(1CF7.3)                                                          SUPCC840
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```
      RETURN                                                                    SUBCC850
      END                                                                       SUBCC860
C0C0015                                                                         SUBCC870
C                                                                               SUBCC880

      SUBROUTINE READ (Z,XC,YC,PHISYM,NOF,IMAX,JMAX,G)                          SUBCC890
C                                                                               SUBCC900
C    READS THE NON-SYMMETRIC PORTION OF THE GARRAY AND EXPANDS IT TO AN         SUBCC910
C    ORTHOGONAL SET. NOTE, INSURE SUFFICIENT DIMENSIONS IN MAIN PROGRAM.        SUBCC920
C                                                                               SUBCC930
      COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM                              SUBCC940
      COMMON /IO/ CMS,IN1,IN2,IN4                                               SUBCC950
      DIMENSION G(IMAX,JMAX)                                                    SUBCC960
      READ (IN1,39) NOF,IMAX,JMAX,ISYM,JSYM,IMS,JMS                             SUBCC970
      READ (IN1,38) Z,XC,YC,PHISYM                                              SUBCC980
      READ (IN1,38) ((G(I,J),I=1,IMS),J=1,JMS)                                  SUBCC990
      WRITE(6,37) NOF,Z,XC,YC,PHISYM,IMAX,JMAX,JSYM                             SUBC1000
      RJMX=JMAX                                                                 SUBC1010
      MSYM=JSYM                                                                 SUBC1020
      IF ((MSYM.EQ.0).OR.(MSYM.GT.JMAX)) MSYM=1                                 SUBC1030
      FCU=ISYM*JSYM*JMAX                                                        SUBC1040
      IF (JSYM.GT.JMAX) FCU=JMAX                                                SUBC1050
      QSYM=FCU/RJMX                                                             SUBC1060
      DO 4 J=1,JMS                                                              SUBC1070
      IF(ISYM.EQ.1) GO TO 2                                                     SUBC1080
      DO 1 I=1,IMS                                                              SUBC1090
      II=IMAX+1-I                                                               SUBC1100
1     G(II,J)=G(I,J)                                                            SUBC1110
      GO TO 4                                                                   SUBC1120
2     J2=JMAX/2+1-J                                                             SUBC1130
      J3=JMAX/2+J                                                               SUBC1140
      J4=JMAX+1-J                                                               SUBC1150
      DO 3 I=1,IMAX                                                             SUBC1160
      II=IMAX+1-I                                                               SUBC1170
      G(I,J2)=G(I,J)                                                            SUBC1180
      G(II,J3)=G(I,J)                                                           SUBC1190
3     G(II,J4)=G(I,J)                                                           SUBC1200
4     CONTINUE                                                                  SUBC1210
39      FORMAT(1CI5)                                                            SUBC1220
38    FORMAT (10F7.3)                                                           SUBC1230
37    FORMAT(///' MODE 3 READS GARRAY DIRECTLY:   NOF='I4//     Z='F7.3SUBC1240
     1,'    XC=',F7.3,'    YC=',F7.3,'    PHISYM=',F7.3//     IMAX=',I4,SUBC1250
     2',    JMAX=',I4,',   JSYM=',I4///)                                       SUBC1260
      RETURN                                                                    SUBC1270
      END                                                                       SUBC1280
C000016                                                                         SUBC1290
C                                                                               SUBC1300
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```
      SUBROUTINE MAP (IM,JM,A,N,Z,BAND)                                  SUBC1310
C                                                                        SUBC1320
CCC   MAP CALLS SUBROUTINE MIMPII AND PLOTS A CONTOUR MAP OF THE ARRAY   SUBC1330
C                                                                        SUBC1340
      DIMENSION A(IM,JM),T(24)                                           SUBC1350
      DATA BL/1H /                                                       SUBC1360
      DO 1 I=1,24                                                        SUBC1370
1     T(I)=BL                                                            SUBC1380
      ICON=1                                                             SUBC1390
      IF(BAND.LT.0.) ICON=0                                              SUBC1400
      IF(BAND.LT.0.) BAND=-BAND                                          SUBC1410
      AMIN=0.                                                            SUBC1420
      IJT=0                                                              SUBC1430
      AZ=1.                                                              SUBC1440
      BZ=0.                                                              SUBC1450
      WRITE(6,49) N,Z                                                    SUBC1460
      CALL MIMPII (A,IM,JM,T,BAND,AZ,BZ,AMIN,IJT,ICON)                   SUBC1470
49    FORMAT (1H1//'  THE FUNCTION SURFACE, TEST NO.'I3,'  Z='F5.3//)    SUBC1480
      RETURN                                                             SUBC1490
      END                                                                SUBC1500
C0000C17                                                                 SUBC1510
C                                                                        SUBC1520

      SUBROUTINE GPLOT (G,GA,JMS)                                        SUBC1530
C                                                                        SUBC1540
CCC   GPLOT PRINTS A ROUGH PLOT OF THE LINE INTEGRAL FUNCTIONS IN GARRAY. SUBC1550
C                                                                        SUBC1560
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z    SUBC1570
      COMMON /TAB/ INDEX(7),JSYM,ISYM                                    SUBC1580
      COMMON /TAB2/ IPT,KPT,LPT,MPT,REST(5)                              SUBC1590
      DIMENSION G(IMAX,JMAX),GA(IMAX,JMAX),ROW(101)                      SUBC1600
      DIMENSION A(201),B(101),C(201),D(101)                              SUBC1610
      JM=101                                                             SUBC1620
      DATA BL,PL,ST,OH,EX/1H ,1H+,1H*,1H-,1HX/                           SUBC1630
      JMS2=JMAX/2+1                                                      SUBC1640
      IF(ISYM.EQ.2)JMS2=1                                                SUBC1650
      JMS3=JMS2+JMS-1                                                    SUBC1660
      DO 8 J=JMS2,JMS3                                                   SUBC1670
      WRITE(6,67) (ST,I=1,120)                                           SUBC1680
      DO 1 I=1,IMAX                                                      SUBC1690
      A(I)=G(I,J)                                                        SUBC1700
1     C(I)=GA(I,J)                                                       SUBC1710
      AS=.5                                                              SUBC1720
      BS=.0                                                              SUBC1730
      CALL INTERP (A,IMAX,AS,B,JM,BS)                                    SUBC1740
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      CALL INTERP (C,IMAX,AS,D,JM,BS)                                   SUBC1750
      WRITE (6,69) J                                                    SUBC1760
      BIG=0.                                                            SUBC1770
      SMALL=0.                                                          SUBC1780
      DO 2 I=1,IMAX                                                     SUBC1790
      IF(A(I).GT.BIG) BIG=A(I)                                          SUBC1800
      IF(C(I).GT.BIG) BIG=C(I)                                          SUBC1810
      IF(A(I).LT.SMALL) SMALL=A(I)                                      SUBC1820
2     IF(C(I).LT.SMALL) SMALL=C(I)                                      SUBC1830
      RANGE=BIG-SMALL                                                   SUBC1840
      RINK=RANGE/80.                                                    SUBC1850
      TOP=BIG+RINK                                                      SUBC1860
      CEN=BIG                                                           SUBC1870
      BOT=BIG-RINK                                                      SUBC1880
      KC=0                                                              SUBC1890
      DO 7 K=1,41                                                       SUBC1900
      IC=0                                                              SUBC1910
      DO 5 I=1,101                                                      SUBC1920
      ROW(I)=BL                                                         SUBC1930
      IF((I.EQ.1).OR.(I.EQ.51).OR.(I.EQ.101))ROW(I)=PL                  SUBC1940
      IF((K.EQ.1).OR.(K.EQ.41)) ROW(I)=PL                               SUBC1950
      IF((TOP.GE.0.).AND.(BOT.LE.0.)) ROW(I)=DH                         SUBC1960
      IF(IC.EQ.5) GO TO 3                                               SUBC1970
      GO TO 4                                                           SUBC1980
3     IC=0                                                              SUBC1990
      IF(KC.EQ.10) ROW(I)=PL                                            SUBC2000
4     IF(KPT.LE.2) GO TO 5                                              SUBC2010
      IF((D(I).LE.TOP).AND.(D(I).GE.BOT)) ROW(I)=ST                     SUBC2020
5     IF((B(I).LE.TOP).AND.(B(I).GE.BOT)) ROW(I)=PX                     SUBC2030
6     IC=IC+1                                                           SUBC2040
      IF(KC.EQ.5) KC=0                                                  SUBC2050
      IF(KC.NE.0) WRITE (6,65) (ROW(I),I=1,101)                         SUBC2060
      IF(KC.EQ.0) WRITE (6,68) CEN,(ROW(I),I=1,101)                     SUBC2070
      TOP = TOP-2.*RINK                                                 SUBC2080
      CEN=CEN-2.*RINK                                                   SUBC2090
      BOT=BOT-2.*RINK                                                   SUBC2100
7     KC=KC+1                                                           SUBC2110
9     WRITE (6,66) (ST,I=1,120)                                         SUBC2120
69    FORMAT (///'      J=',I3///)                                      SUBC2130
68    FORMAT (1X,F8.3,1X,101A1)                                         SUBC2140
67    FORMAT (1H1,//121A1//)                                            SUBC2150
66    FORMAT (///121A1///)                                              SUBC2160
65    FORMAT (10X,101A1)                                                SUBC2170
      RETURN                                                            SUBC2180
      END                                                               SUBC2190
CCCCCC                                                                  SUBC2200
C                                                                       SUBC2210
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      SUBROUTINE INTERP (A,IM,AS,B,JM,BS)                                   SUBC2220
                                                                            SUBC2230
C     INTERP CONVERTS A REGULAR VECTOR A OF IM POINTS TO A REGULAR VECTOR   SUBC2240
C     B OF JM POINTS.  OS=.5 FOR A VECTOR WITH POINTS DEFINED IN THE        SUBC2250
C     CENTER OF THE INTERVAL, AS AND BS ARE THE % OF AN INTERVAL FROM THE   SUBC2260
C     EDGE OF THE FIELD TO THE FIRST POINT (.0 OR .5 FOR EDGE OR CENTER     SUBC2270
C     DEFINED POINTS)                                                       SUBC2280
C                                                                           SUBC2290
      DIMENSION A(IM),B(JM)                                                 SUBC2300
      RIM=IM                                                                SUBC2310
      RJM=JM                                                                SUBC2320
      RAT=(RIM-1.+2.*AS)/(RJM-1.+2.*BS)                                     SUBC2330
      DO  2  I=1,JM                                                         SUBC2340
      RI=I                                                                  SUBC2350
      AI=RAT*(RI+BS)-AS                                                     SUBC2360
      IA=AI                                                                 SUBC2370
      F=AI-FLOAT(IA)                                                        SUBC2380
      IF((IA.EQ.0).OR.(IA.EQ.JM)) GO TO 1                                   SUBC2390
      B(I)=A(IA)+F*(A(IA+1)-A(IA))                                          SUBC2400
      GO TO 2                                                               SUBC2410
1     IF(IA.EQ.0) B(I)=A(1)*(F-AS)/(1.-AS)                                  SUBC2420
      IF(IA.EQ.JM) B(I)=A(JM)*F/(1.-AS)                                     SUBC2430
2     CONTINUE                                                              SUBC2440
      RETURN                                                                SUBC2450
      END                                                                   SUBC2460
C                                                                           SUBC2470
                                                                            SUBC2480
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CCCCCCC                                                                     SPLC0010
C     ------------------------------------------------------------------    SPLC0020
C                                                                           SPLC0030
C        SUBROUTINE SPLINE                                                  SPLC0040
C                                                                           SPLC0050
C        PURPOSE                                                            SPLC0060
C            PROVIDES INTERPOLATED VALUE USING "CUBIC SPLINE FITTING"       SPLC0070
C                                                                           SPLC0080
C        USAGE                                                              SPLC0090
C            FIRST CALL TO SUBROUTINE:                                      SPLC0100
C            CALL SPLINE(X,Y,M,XINT,YINT)                                   SPLC0110
C                                                                           SPLC0120
C            SUBSEQUENT CALLS:                                              SPLC0130
C            CALL SPLINN(X,Y,M,XINT,YINT)                                   SPLC0140
C                                                                           
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C        DESCRIPTION OF PARAMETERS                                          SPLCC150
CCCCCCCC     X:     MONOTONICALLY INCREASING ABSCISSA ARRAY                 SPLCC160
CCCCCCCC     Y:     ONE-FOR-ONE CORRESPONDING ORDINATE ARRAY                SPLCC170
CCCCCCCC     M:     NUMBER OF X AND Y VALUES SUPPLIED < OR = 300            SPLCC180
CCCCCCCC     XINT:  VALUE OF ABSCISSA FOR WHICH CORRESPONDING ORDINATE      SPLCC190
CCCCCCCC            IS TO BE INTERPOLATED (OR EXTRAPOLATED)                 SPLCC200
CCCCCCCC     YINT:  INTERPOLATED (OR EXTRAPOLATED) ORDINATE VALUE           SPLCC210
CCCCCCCC                                                                    SPLCC220
CCCCCCCC                                                                    SPLCC240
CCCCCCCC     REMARKS                                                        SPLCC250
CCCCCCCC         IF SPECIFIED X FALLS OUTSIDE OF RANGE, AN EXTRAPOLATED     SPLCC260
CCCCCCCC         VALUE WILL BE SUPPLIED                                     SPLCC270
CCCCCCCC                                                                    SPLCC280
CCCCCCCC     SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  SPLCC290
CCCCCCCC         SUBROUTINE SPLICO IS INCLUDED IN SUBROUTINE SPLIN PACKAGE  SPLCC300
CCCCCCCC                                                                    SPLCC310
CCCCCCCC     MATHEMATICAL METHOD                                            SPLCC320
CCCCCCCC         UPON FIRST ENTRY TO SPLIN, A CALL TO SPLICO IS MADE TO     SPLCC330
CCCCCCCC         DETERMINE THE COEFFICIENTS TO BE USED IN PERFORMING THE    SPLCC340
CCCCCCCC         INTERPOLATIONS. SEARCH FOR BRACKETING ABSCISSA VALUES IS   SPLCC350
CCCCCCCC         ALWAYS MADE FROM THE REFERENCE LAST USED IN INTERPOLATING. SPLCC360
CCCCCCCC                                                                    SPLCC370
CCCCCCCC     REFERENCE                                                      SPLCC380
CCCCCCCC         BENNINGTON, RALPH H.,"INTRODUCTORY COMPUTER METHODS AND     SPLCC390
CCCCCCCC         NUMERICAL ANALYSIS", THE MACMILLAN COMPANY, NEW YORK, 1965 SPLCC400
CCCCCCCC                                                                    SPLCC410
CCCCC    ------------------------------------------------------------------SPLCC420
C                                                                           SPLCC430

      SUBROUTINE SPLINE(X,Y,M,XINT,YINT)                                    SPLCC440
      DIMENSION X(M),Y(M),C(4,300)                                          SPLCC460
      CALL SPLICO(X,Y,M,C)                                                  SPLCC470
      K=1                                                                   SPLCC480
      ENTRY SPLINN(X,Y,M,XINT,YINT)                                         SPLCC490
    3 IF(XINT-X(1)) 70,1,2                                                  SPLCC500
   70 K=1                                                                   SPLCC510
      GO TO 7                                                               SPLCC520
    1 YINT=Y(1)                                                             SPLCC530
      RETURN                                                                SPLCC540
    2 IF(XINT-X(K+1))6,4,5                                                  SPLCC550
    4 YINT=Y(K+1)                                                           SPLCC560
      RETURN                                                                SPLCC570
    5 K=K+1                                                                 SPLCC580
      IF(M-K) 71,71,3                                                       SPLCC590
   71 K=M-1                                                                 SPLCC600
      GO TO 7                                                               SPLCC610
    6 IF(XINT-X(K))13,12,11                                                 SPLCC620
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   12 YINT=Y(K)                                                                    SPLC0620
      RETURN                                                                       SPLC0630
   13 K=K-1                                                                        SPLC0640
      GO TO 6                                                                      SPLC0650
    7 PRINT 101,XINT                                                               SPLC0660
  101 FORMAT(8HCXINT = E18.9,32H, OUT OF RANGE FOR INTERPOLATION)                  SPLC0670
   11 YINT=(X(K+1)-XINT)*(C(1,K)*(X(K+1)-XINT)**2+C(3,K))                          SPLC0680
      YINT=YINT+(XINT-X(K))*(C(2,K)*(XINT-X(K))**2+C(4,K))                         SPLC0690
      RETURN                                                                       SPLC0700
      END                                                                          SPLC0710
                                                                                   SPLC0720

      SUBROUTINE SPLICO(X,Y,M,C)                                                   SPLC0730
      DIMENSION X(M),Y(M),C(4,300),D(300),P(300),E(300),A(300,3),B(300),SPLC0740
     17(300)                                                                       SPLC0750
      MM=M-1                                                                       SPLC0760
      DO 2 K=1,MM                                                                  SPLC0770
      D(K)=X(K+1)-X(K)                                                             SPLC0780
      P(K)=D(K)/6.                                                                 SPLC0790
    2 E(K)=(Y(K+1)-Y(K))/D(K)                                                      SPLC0800
      DO 3 K=2,MM                                                                  SPLC0810
    3 B(K)=E(K)-E(K-1)                                                             SPLC0820
      A(1,2)=-1.-D(1)/D(2)                                                         SPLC0830
      A(1,3)=D(1)/D(2)                                                             SPLC0840
      A(2,3)=P(2)-P(1)*A(1,3)                                                      SPLC0850
      A(2,2)=2.*(P(1)+P(2))-P(1)*A(1,2)                                            SPLC0860
      A(2,3)=A(2,3)/A(2,2)                                                         SPLC0870
      B(2)=B(2)/A(2,2)                                                             SPLC0880
      DO 4 K=3,MM                                                                  SPLC0890
      A(K,2)=2.*(P(K-1)+P(K))-P(K-1)*A(K-1,3)                                      SPLC0900
      B(K)=B(K)-P(K-1)*B(K-1)                                                      SPLC0910
      A(K,3)=P(K)/A(K,2)                                                           SPLC0920
    4 B(K)=B(K)/A(K,2)                                                             SPLC0930
      Q=D(M-2)/D(M-1)                                                              SPLC0940
      A(M,1)=1.+Q+A(M-2,3)                                                         SPLC0950
      A(M,2)=-Q-A(M,1)*A(M-1,3)                                                    SPLC0960
      B(M)=B(M-2)-A(M,1)*B(M-1)                                                    SPLC0970
      Z(M)=B(M)/A(M,2)                                                             SPLC0980
      MN=M-2                                                                       SPLC0990
      DO 6 I=1,MN                                                                  SPLC1000
      K=M-I                                                                        SPLC1010
    6 Z(K)=B(K)-A(K,3)*Z(K+1)                                                      SPLC1020
      Z(1)=-A(1,2)*Z(2)-A(1,3)*Z(3)                                                SPLC1030
      DO 7 K=1,MM                                                                  SPLC1040
      Q=1./(6.*D(K))                                                               SPLC1050
      C(1,K)=Z(K)*Q                                                                SPLC1060
      C(2,K)=Z(K+1)*Q                                                              SPLC1070
      C(3,K)=Y(K)/D(K)-Z(K)*P(K)                                                   SPLC1080
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    7 C(4,K)=Y(K+1)/D(K)-Z(K+1)*P(K)                                        SPL01100
      RETURN                                                                SPL01110
      END                                                                   SPL01120

CCCCC2C
C                                                                           METOCC10
C     ***************************************************************     METOCC20
C                                                                           METOCC30
C     SUBROUTINE MTMPII                                                     METOCC40
C                                                                           METOCC50
C     PURPOSE                                                               METOCC60
C               MTMPII WILL PRODUCE,ON THE PRINTER, A CONTOUR MAP           METOCC70
C               OF ANY SINGLE PRECISION TWO DIMENSIONAL ARRAY.              METOCC80
C                                                                           METOCC90
C     USAGE                                                                 METOC100
C               CALL MTMPII(Y,N,M,T,BND,AZ,BZ,AMIN,IJT,ICON)                METOC110
C                                                                           METOC120
C     DESCRIPTION OF PARAMETERS                                             METOC130
C                                                                           METOC140
C               Y     - THE ARRAY TO BE CONTOURED. DIMENSIONED Y(N,M)       METOC150
C               N     - NUMBER OF ROWS IN Y.                                METOC160
C               M     - NUMBER OF COLUMNS IN Y.                             METOC170
C               T     - ARRAY FOR PLOT TITLE. REAL*4 T(24).                 METOC180
C               BND   - BANDWIDTH FOR THE CONTOURING. IF BND IS ZERO        METOC190
C                       A BANDWIDTH WILL BE CALCULATED AS FOLLOWS           METOC200
C                       BND=(MAX(Y)-MIN(Y))/15.                             METOC210
C               AZ    - A LINEAR TRANSFORMATION MAYBE PERFORMED ON          METOC220
C                       THE ARRAY Y OF THE FOLLOWING FORM AZ*Y+BZ.          METOC230
C                       IF AZ=0 THEN AZ WILL BE COMPUTED SUCH THAT          METOC240
C                       MAX(|MAX(Y)|,|MIN(Y)|) WILL BE LESS THAN 1,         METOC250
C                       AND BZ WILL BE LEFT AS INPUT.                       METOC260
C               BZ    - SEE UNDER AZ                                        METOC270
C               AMIN  - THE LEVEL AT WHICH CONTOURING WILL BEGIN. IF        METOC280
C                       AMIN > MIN(Y) THEN AMIN WILL BE CALCULATED          METOC290
C                       TO BE ,WITH REFERENCE AT ZERO, THE NEXT LOWER       METOC300
C                       CONTOUR LEVEL FROM MIN(Y) AS DETERMINED BY          METOC310
C                       BND.                                                METOC320
C               IJT   - IF IJT=0 AMIN WILL BE CALCULATED AS DESCRIBED       METOC330
C                       ABOVE.                                              METOC340
C               ICON  - IF ICON=0 NO CONTOURING WILL BE DONE BUT THE        METOC350
C                       ARRAY Y WILL BE PRINTED IN THE PLOT FORMAT.          METOC360
C                                                                           METOC370
C     REMARKS                                                               METOC380
C               MTMPII REQUIRES A PRINTER WITH 132 PRINT POSITIONS.         METOC390
C               IF NECESSARY THE MAP WILL BE SEGMENTED COLUMNWISE.          METOC400
C               THE ROWS AND COLUMNS ARE NUMBERED ALONG THE EDGES SO        METOC410
C               THAT A SEGMENTED MAP MAYBE EASILY JOINED TOGETHER.          METOC420
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C                   ONLY THREE SIGNIFICANT FIGURES WILL BE PRINTED AT    METCC430
C                   EACH POINT. THE POSITION OF THE FIRST SIGNIFICANT    METCC440
C                   DIGIT WILL BE DETERMINED BY MAX(|MAX(Y)|,|MIN(Y)|).  METCC450
C                   THE PLOT WILL BE PRODUCED ON A INCH INCH GRID. IT    METCC460
C                   WILL BE ASSUMED THAT THE SPACING BETWEEN POINTS IN   METCC470
C                   BOTH DIRECTIONS IS THE SAME AND EQUAL FOR ALL POINTS METCC480
C                                                                        METCC490
C         SUBROUTINES REQUIRED                                           METCC500
C                   NONE                                                 METCC510
C                                                                        METCC520
C         METHOD                                                         METCC530
C                   THE CONTOUR LEVELS ARE DETERMINED BY SIMPLE LINEAR   METCC540
C                   INTERPOLATION FROM THE FOUR SURROUNDING POINTS.      METCC550
C                                                                        METCC560
C         ***************************************************************METCC570
C                                                                        METCC580
C                                                                        METCC590
C   MTMPII SUBROUTINE FOR ONE-INCH GRID SPACING                          METCC600
C   OAKES CODE 5105    15 JAN 69                                         METCC610
C                                                                        METCC620
C
      SUBROUTINE MTMPII(Y,N,M,T,BND,AZ,BZ,AMIN,IJT,ICON)                 METCC630
      REAL*4 IH,KG,ITJZ                                                  METCC640
      DIMENSION A(140),B(140),C(140),D(140),IH(20),Y(N,M),TP(10),TPX(10)METCC650
     Z   ,TPM(10),XMT(10),BTM(10),BTX(10),BT(10),KG(10),T(24)            METCC660
      DIMENSION E(140),F(140),G(140),H(140)                              METCC670
C                                                                        METCC680
C                                                                        METCC690
      DATA DLE/4H    /,EPL/4H+   /,EMI/4H-   /,IH/1H0,1H ,1H1,1H ,1H2,  METCC700
     1IH ,1H3,1H ,1H4,1H ,1H5,1H ,1H6,1H ,1H7,1H ,1H8,1H ,1H9,1H /,KG/   METCC710
     2IHC,1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9/,BLK/4H    /              METCC720
C                                                                        METCC730
      YMIN=Y(1,1)                                                        METCC740
      YMAX=Y(1,1)                                                        METCC750
      DO 2C I=1,M                                                        METCC760
      DO 1C J=1,N                                                        METCC770
      YMIN=AMIN1(YMIN,Y(J,I))                                            METCC780
      YMAX=AMAX1(YMAX,Y(J,I))                                            METCC790
   1C CONTINUE                                                           METCC800
   2C CONTINUE                                                           METCC810
      DELY=YMAX-YMIN                                                     METCC820
      IF(BND) 25,25,3C                                                   METCC830
   25 BND=DELY/15.C                                                      METCC840
   3C IF (AMIN-YMIN) 31,31,32                                            METCC850
   31 IF (IJT) 33,32,33                                                  METCC860
   32 PD=YMIN/BND                                                        METCC870
      PE=ABS(PD-INT(PD))                                                 METCC880
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      IF (YMIN) 2,1,1                                                   METCC890
    1 AMIN=YMIN-PF*BND                                                  METCC900
      GO TO 33                                                          METCC910
    2 AMIN=YMIN-(1.0-PF)*BND                                            METCC920
   33 AHLD=AZ                                                           METCC930
      IF(AZ) 55,35,55                                                   METCC940
   35 SM=AMAX1(ABS(YMIN),ABS(YMAX))                                     METCC950
      NS=0                                                              METCC960
   40 NS=NS+1                                                           METCC970
      SM=10.0*SM                                                        METCC980
      IF(SM-1.0)40,50,45                                                METCC990
   45 NS=NS-1                                                           METC1000
      SM=SM/10.0                                                        METC1010
      IF(SM-1.0)50,50,45                                                METC1020
   50 AHLD=10.0**NS                                                     METC1030
   55 BAND=BND/2.0                                                      METC1040
      PRINT 70                                                          METC1050
      PRINT 6,T                                                         METC1060
    6 FORMAT(5X,2+A4,//)                                                METC1070
      PRINT 57,AHLD,BZ                                                  METC1080
   57 FORMAT(1HC,65HTHE FOLLOWING TRANSFORMATION WAS PERFORMED ON THE IN METC1090
     1PUT MATRIX      /5X,1H(,E12.5,8H*Y(I,J)+,E12.5,1H) //2X,73HAND THREMETC1100
     2 DIGITS TO THE RIGHT OF THE DECIMAL POINT ARE PRINTED IN THE MAP )METC1110
C                                                                       METC1120
      PRINT 54,YMAX,YMIN                                                METC1130
   54 FORMAT(/4X,5HYMAX=,F15.7,5X,5HYMIN=,E15.7)                        METC1140
      IF (ICON)5,58,5                                                   METC1150
    5 PRINT 11,BND                                                      METC1160
   11 FORMAT(2X,17HTHE BAND WIDTH IS,E12.5,6H UNITS //4X,14HCONTOUR LEVEMETC1170
     1LS          )                                                     METC1180
      I=0                                                               METC1190
      YTOP=AMIN                                                         METC1200
      IF(ABS(YMIN-YMAX)-100.0*BND)53,53,58                              METC1210
   53 YB=YTOP                                                           METC1220
      YTOP=YTOP+BND                                                     METC1230
      I=I+1                                                             METC1240
      J=MOD(I,20)                                                       METC1250
      ITJZ=IH(J)                                                        METC1260
      IF(YB-YMAX)59,58,58                                               METC1270
   59 PRINT 61,YB,YTOP,ITJZ                                             METC1280
   61 FORMAT(/4X,E10.3,4H TO ,E10.3,2H =,1X,A1)                         METC1290
      GO TO 53                                                          METC1300
   58 NCOP=0                                                            METC1310
      NCP=0                                                             METC1320
   50 PRINT 70                                                          METC1330
   70 FORMAT(1H1)                                                       METC1340
      PRINT 6,T                                                         METC1350
      NLINE=0                                                           METC1360
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      NCCP=NCP+1                                                        METC1370
      NCP = NCP + 13                                                    METC1380
   73 IF(NCP-M)8C,8C,75                                                 METC1390
   75 NCP=M                                                             METC1400
   8C CONTINUE                                                          METC1410
      J=-2                                                              METC1420
      NLINE=NLINE+1                                                     METC1430
      LLINE=N-NLINE+1                                                   METC1440
C SET UP HEADING                                                        METC1450
      IF(NCCP-1) 85,85,9C                                               METC1460
   85 J = - 1                                                           METC1470
   9C DO 1CC I = 1,135                                                  METC1480
      A(I)=BLK                                                          METC1490
      B(I)=BLK                                                          METC1500
      F(I) = BLK                                                        METC1510
  1CC CONTINUE                                                          METC1520
  11C DO 16C L=NCCP,NCP                                                 METC1530
      J = J+8                                                           METC1540
      KI=L                                                              METC1550
      IF(KI-1CC) 13C,12C,12C                                            METC1560
  12C LL=KI/1CC                                                         METC1570
      A(J)=KC(LL+1)                                                     METC1580
      KI=KI-1CC*LL                                                      METC1590
      GO TO 135                                                         METC1600
  13C A(J)=KC(1)                                                        METC1610
  135 J=J+1                                                             METC1620
      IF(KI-1C) 15C,14C,14C                                             METC1630
  14C LL=KI/1C                                                          METC1640
      A(J)=KC(LL+1)                                                     METC1650
      KI=KI-1C*LL                                                       METC1660
      GO TO 155                                                         METC1670
  15C A(J)=KC(1)                                                        METC1680
  155 J=J+1                                                             METC1690
      A(J)=KC(KI+1)                                                     METC1700
  16C CONTINUE                                                          METC1710
C SETUP FIRST ROW OF ARRAY                                              METC1720
      GO TO 26C                                                         METC1730
  17C NLINE=NLINE+1                                                     METC1740
      LLINE=N-NLINE+1                                                   METC1750
      IF(NLINE-N) 18C,18C,38C                                           METC1760
  18C DO 19C I=1,135                                                    METC1770
      A(I)=BLK                                                          METC1780
      B(I)=BLK                                                          METC1790
      C(I)=BLK                                                          METC1800
      D(I)=BLK                                                          METC1810
      E(I) = BLK                                                        METC1820
      F(I) = BLK                                                        METC1830
      G(I) = BLK                                                        METC1840
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      F(I) = BLK                                                        METC1850
  190 CONTINUE                                                          METC1860
      IF (ICON)195,260,195                                              METC1870
  195 NCY=NCOP-1                                                        METC1880
      J=4                                                               METC1890
      IF(NCY)200,200,210                                                METC1900
  200 J=5                                                               METC1910
  210 NCY=NCY+1                                                         METC1920
      IF(NCY-NCP) 220,220,260                                           METC1930
  220 IF(NCY-M) 230,260,260                                             METC1940
  230 NLINE = NLINE - 1                                                 METC1950
      YD1 = Y(NLINE,NCY) - Y(NLINE+1,NCY)                               METC1960
      YD2=Y(NLINE,NCY+1)-Y(NLINE+1,NCY+1)                               METC1970
      TP(1) = Y(NLINE,NCY)-0.125*YD1                                    METC1980
      TPX(1)=Y(NLINE,NCY)-0.250*YD1                                     METC1990
      TPM(1)=Y(NLINE,NCY)-0.375*YD1                                     METC2000
      XMT(1)=Y(NLINE,NCY)-0.500*YD1                                     METC2010
      BTM(1)=Y(NLINE,NCY)-0.625*YD1                                     METC2020
      BTX(1)=Y(NLINE,NCY)-0.750*YD1                                     METC2030
      BT (1)=Y(NLINE,NCY)-0.875*YD1                                     METC2040
      TP (10)=Y(NLINE,NCY+1)-0.125*YD2                                  METC2050
      TPX(10)=Y(NLINE,NCY+1)-0.250*YD2                                  METC2060
      TPM(10)=Y(NLINE,NCY+1)-0.375*YD2                                  METC2070
      XMT(10)=Y(NLINE,NCY+1)-0.500*YD2                                  METC2080
      BTM(10)=Y(NLINE,NCY+1)-0.625*YD2                                  METC2090
      BTX(10)=Y(NLINE,NCY+1)-0.750*YD2                                  METC2100
      BT (10)=Y(NLINE,NCY+1)-0.875*YD2                                  METC2110
      NLINE = NLINE + 1                                                 METC2120
      D1=0.1 * (TP (10)-TP (1))                                         METC2130
      D2=0.1 * (TPX(10)-TPX(1))                                         METC2140
      D3=0.1 * (TPM(10)-TPM(1))                                         METC2150
      D4=0.1 * (XMT(10)-XMT(1))                                         METC2160
      D5=0.1 * (BTM(10)-BTM(1))                                         METC2170
      D6=0.1 * (BTX(10)-BTX(1))                                         METC2180
      D7=0.1 * (BT (10)-BT (1))                                         METC2190
      DO 240 I = 2,9                                                    METC2200
      TP (I)=TP (I-1)+D1                                                METC2210
      TPX(I)=TPX(I-1)+D2                                                METC2220
      TPM(I)=TPM(I-1)+D3                                                METC2230
      XMT(I)=XMT(I-1)+D4                                                METC2240
      BTM(I)=BTM(I-1)+D5                                                METC2250
      BTX(I)=BTX(I-1)+D6                                                METC2260
      BT (I)=BT (I-1)+D7                                                METC2270
  240 CONTINUE                                                          METC2280
      DO 250 I=1,10                                                     METC2290
      J=J+1                                                             METC2300
      I1=MOD(IFIX((TP (I)-AMIN)/BND),20)+1                              METC2310
      I2=MOD(IFIX((TPX(I)-AMIN)/BND),20)+1                              METC2320
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      I3=MOD(IFIX((TPM(I)-AMIN)/BND),20)+1                              MET02330
      I4=MOD(IFIX((XMT(I)-AMIN)/BND),20)+1                              MET02340
      I5=MOD(IFIX((BTM(I)-AMIN)/BND),20)+1                              MET02350
      I6=MOD(IFIX((BTX(I)-AMIN)/BND),20)+1                              MET02360
      I7=MOD(IFIX((BT (I)-AMIN)/BND),20)+1                              MET02370
      A(J)=IH(I1)                                                       MET02380
      B(J)=IH(I2)                                                       MET02390
      C(J)=IH(I3)                                                       MET02400
      D(J)=IH(I4)                                                       MET02410
      E(J)=IH(I5)                                                       MET02420
      F(J)=IH(I6)                                                       MET02430
      G(J)=IH(I7)                                                       MET02440
  250 CONTINUE                                                          MET02450
      GO TO 210                                                         MET02460
  260 NCY=NCOP-1                                                        MET02470
      J=-2                                                              MET02480
      IF(NCY) 265,265,270                                               MET02490
  265 J=-1                                                              MET02500
      GO TO 330                                                         MET02510
  270 NCY=NCY+1                                                         MET02520
      IF(NCY-NCP) 280,280,310                                           MET02530
  280 J=J+7                                                             MET02540
      THLD=AHLD*Y(NLINE,NCY)+BZ                                         MET02550
      IF(THLD) 285,290,290                                              MET02560
  285 H(J)=BMI                                                          MET02570
      GO TO 295                                                         MET02580
  290 H(J)=BPL                                                          MET02590
  295 NUM=INT(ABS(THLD-INT(THLD))*1000.0+0.5)                           MET02600
      NDS=100                                                           MET02610
      DO 300 KK=1,3                                                     MET02620
      J=J+1                                                             MET02630
      KI=NUM/NDS                                                        MET02640
      H(J)=KG(KI+1)                                                     MET02650
      NUM=NUM-KI*NDS                                                    MET02660
      NDS=NDS/10                                                        MET02670
  300 CONTINUE                                                          MET02680
      GO TO 270                                                         MET02690
  310 IF(NCP-M) 360,320,320                                             MET02700
  320 IF(J-127)330,330,360                                              MET02710
  330 J=J+3                                                             MET02720
      KI=NLINE                                                          MET02730
      IF(KI-100) 340,335,335                                            MET02740
  335 LL=KI/100                                                         MET02750
      H(J)=KG(LL+1)                                                     MET02760
      KI=KI-100*LL                                                      MET02770
      GO TO 343                                                         MET02780
  340 H(J)=KG(1)                                                        MET02790
  343 J=J+1                                                             MET02800
```

```
      IF(KI-1C) 35C,345,345                                                    METO2810
345   LL=KI/1C                                                                 METO2820
      F(J)=KG(LL+1)                                                            METO2830
      KI=KI-1C*LL                                                              METO2840
      GO TO 355                                                                METO2850
35C   F(J)=KG(1)                                                               METO2860
355   J=J+1                                                                    METO2870
      F(J)=KG(KI+1)                                                            METO2880
      J=J-5                                                                    METO2890
      IF(NCY-1) 27C,27C,36C                                                    METO2900
36C   IF(NLINE-1)362,362,368                                                   METO2910
362   PRINT 37C,(A(I),I=1,132),(B(IP1),IP1=1,132),(H(IP2),IP2=1,132)           METO2920
      GO TO 17C                                                                METO2930
368   PRINT 37C,(A(I),I=1,132),(B(IP1),IP1=1,132),(C(IP2),IP2=1,132),          METO2940
     1(D(IP3),IP3=1,132),(E(IP4),IP4=1,132),(F(IP5),IP5=1,132),                METO2950
     2(G(IP6),IP6=1,132),(H(IP7),IP7=1,132)                                    METO2960
37C   FORMAT(132A1)                                                            METO2970
      GO TO 17C                                                                METO2980
38C   DO 39C I=1,135                                                           METO2990
      A(I)=PLK                                                                 METO3000
      B(I)=BLK                                                                 METO3010
      C(I)=BLK                                                                 METO3020
      D(I)=BLK                                                                 METO3030
39C   CONTINUE                                                                 METO3040
      J=-2                                                                     METO3050
      IF(NCCP-1) 395,395,4CC                                                   METO3060
395   J=-1                                                                     METO3070
4CC   DO 43C L=NCCP,NCP                                                        METO3080
      J=J+3                                                                    METO3090
      KI=L                                                                     METO3100
      IF(KI-1CC) 41C,4C5,4C5                                                   METO3110
4C5   LL=KI/1CC                                                                METO3120
      C(J)=KG(LL+1)                                                            METO3130
      KI=KI-1CC*LL                                                             METO3140
      GO TO 412                                                                METO3150
41C   C(J)=KG(1)                                                               METO3160
412   J=J+1                                                                    METO3170
      IF(KI-1C) 42C,415,415                                                    METO3180
415   LL=KI/1C                                                                 METO3190
      C(J)=KG(LL+1)                                                            METO3200
      KI=KI-1C*LL                                                              METO3210
      GO TO 422                                                                METO3220
42C   C(J)=KG(1)                                                               METO3230
422   J=J+1                                                                    METO3240
      C(J)=KG(KI+1)                                                            METO3250
43C   CONTINUE                                                                 METO3260
      PRINT 37C, (B(IP1),IP1=1,132),(C(IP2),IP2=1,132)                         METO3270
      IF(NCP-M)6C,5CC,5CC                                                      METO3280
```

```
      500 RETURN                                                        METO3290
          END                                                           METO3300

CCCCC21
C                                                                       GAUS   10
C     ..................................................................GAUS   20
C                                                                       GAUS   30
C        SUBROUTINE GAUSS                                               GAUS   40
C                                                                       GAUS   50
C        PURPOSE                                                        GAUS   60
C           COMPUTES A NORMALLY DISTRIBUTED RANDOM NUMBER WITH A GIVEN  GAUS   70
C           MEAN AND STANDARD DEVIATION                                 GAUS   80
C                                                                       GAUS   90
C        USAGE                                                          GAUS  100
C           CALL GAUSS(IX,S,AM,V)                                       GAUS  110
C                                                                       GAUS  120
C        DESCRIPTION OF PARAMETERS                                      GAUS  130
C           IX -IX MUST CONTAIN AN ODD INTEGER NUMBER WITH NINE OR      GAUS  140
C               LESS DIGITS ON THE FIRST ENTRY TO GAUSS. THEREAFTER     GAUS  150
C               IT WILL CONTAIN A UNIFORMLY DISTRIBUTED INTEGER RANDOM  GAUS  160
C               NUMBER GENERATED BY THE SUBROUTINE FOR USE ON THE NEXT  GAUS  170
C               ENTRY TO THE SUBROUTINE.                                GAUS  180
C           S  -THE DESIRED STANDARD DEVIATION OF THE NORMAL            GAUS  190
C               DISTRIBUTION.                                           GAUS  200
C           AM -THE DESIRED MEAN OF THE NORMAL DISTRIBUTION             GAUS  210
C           V  -THE VALUE OF THE COMPUTED NORMAL RANDOM VARIABLE        GAUS  220
C                                                                       GAUS  230
C        REMARKS                                                        GAUS  240
C           THIS SUBROUTINE USES RANDU WHICH IS MACHINE SPECIFIC        GAUS  250
C                                                                       GAUS  260
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  GAUS  270
C           RANDU                                                       GAUS  280
C                                                                       GAUS  290
C        METHOD                                                         GAUS  300
C           USES 12 UNIFORM RANDOM NUMBERS TO COMPUTE NORMAL RANDOM     GAUS  310
C           NUMBERS BY CENTRAL LIMIT THEOREM. THE RESULT IS THEN        GAUS  320
C           ADJUSTED TO MATCH THE GIVEN MEAN AND STANDARD DEVIATION.    GAUS  330
C           THE UNIFORM RANDOM NUMBERS COMPUTED WITHIN THE SUBROUTINE   GAUS  340
C           ARE FOUND BY THE POWER RESIDUE METHOD.                      GAUS  350
C                                                                       GAUS  360
C     ..................................................................GAUS  370
C                                                                       GAUS  380
```

```
      SUBROUTINE GAUSS(IX,S,AM,V)                                       GAUS  390
      A=0.0                                                             GAUS  400
      DO 5C I=1,12                                                      GAUS  410
      CALL RANDU(IX,IY,Y)                                               GAUS  420
      IX=IY                                                             GAUS  430
   5C A=A+Y                                                             GAUS  440
      V=(A-6.C)*S+AM                                                    GAUS  450
      RETURN                                                            GAUS  460
      END                                                               GAUS  470

CCCCC22
C                                                                       RAND   10
C     ..................................................................RAND   20
C                                                                       RAND   30
C        SUBROUTINE RANDU                                               RAND   40
C                                                                       RAND   50
C        PURPOSE                                                        RAND   60
C           COMPUTES UNIFORMLY DISTRIBUTED RANDOM REAL NUMBERS BETWEEN  RAND   70
C           C AND 1.0 AND RANDOM INTEGERS BETWEEN ZERO AND              RAND   80
C           2**31. EACH ENTRY USES AS INPUT AN INTEGER RANDOM NUMBER    RAND   90
C           AND PRODUCES A NEW INTEGER AND REAL RANDOM NUMBER.          RAND  100
C                                                                       RAND  110
C        USAGE                                                          RAND  120
C           CALL RANDU(IX,IY,YFL)                                       RAND  130
C                                                                       RAND  140
C        DESCRIPTION OF PARAMETERS                                      RAND  150
C           IX - FOR THE FIRST ENTRY THIS MUST CONTAIN ANY ODD INTEGER  RAND  160
C                NUMBER WITH NINE OR LESS DIGITS. AFTER THE FIRST ENTRY,RAND  170
C                IX SHOULD BE THE PREVIOUS VALUE OF IY COMPUTED BY THIS RAND  180
C                SUBROUTINE.                                            RAND  190
C           IY - A RESULTANT INTEGER RANDOM NUMBER REQUIRED FOR THE NEXTRAND  200
C                ENTRY TO THIS SUBROUTINE. THE RANGE OF THIS NUMBER IS  RAND  210
C                BETWEEN ZERO AND 2**31                                 RAND  220
C           YFL- THE RESULTANT UNIFORMLY DISTRIBUTED, FLOATING POINT,   RAND  230
C                RANDOM NUMBER IN THE RANGE 0 TO 1.0                    RAND  240
C                                                                       RAND  250
C        REMARKS                                                        RAND  260
C           THIS SUBROUTINE IS SPECIFIC TO SYSTEM/360 AND WILL PRODUCE  RAND  270
C           2**29 TERMS BEFORE REPEATING.  THE REFERENCE BELOW DISCUSSESRAND  280
C           SEEDS (65539 HERE), RUN PROBLEMS, AND PROBLEMS CONCERNING   RAND  290
C           RANDOM DIGITS USING THIS GENERATION SCHEME.  MACLAREN AND   RAND  300
C           MARSAGLIA, JACM 12, P. 83-89, DISCUSS CONGRUENTIAL          RAND  310
C           GENERATION METHODS AND TESTS.  THE USE OF TWO GENERATORS OF RAND  320
C           THE RANDU TYPE, ONE FILLING A TABLE AND ONE PICKING FROM THERAND  330
C           TABLE, IS OF BENEFIT IN SOME CASES.  65539 HAS BEEN        RAND  340
C           SUGGESTED AS A SEED WHICH HAS BETTER STATISTICAL PROPERTIES RAND  350
C           FOR HIGH ORDER BITS OF THE GENERATED DEVIATE.              RAND  360
```

```
C              SEEDS SHOULD BE CHOSEN IN ACCORDANCE WITH THE DISCUSSION     RAND 370
C              GIVEN IN THE REFERENCE BELOW.  ALSO, IT SHOULD BE NOTED THAT RAND 380
C              IF FLOATING POINT RANDOM NUMBERS ARE DESIRED,AS ARE         RAND 390
C              AVAILABLE FROM RANDU, THE RANDOM CHARACTERISTICS OF THE      RAND 400
C              FLOATING POINT DEVIATES ARE MODIFIED AND IN FACT THESE       RAND 410
C              DEVIATES HAVE HIGH PROBABILITY OF HAVING A TRAILING LOW      RAND 420
C              ORDER ZERO BIT IN THEIR FRACTIONAL PART.                     RAND 430
C                                                                           RAND 440
C          SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                    RAND 450
C             NONE                                                          RAND 460
C                                                                           RAND 470
C          METHOD                                                           RAND 480
C             POWER RESIDUE METHOD DISCUSSED IN IBM MANUAL C20-8011,        RAND 490
C             RANDOM NUMBER GENERATION AND TESTING                          RAND 500
C                                                                           RAND 510
C     ..................................................................... RAND 520
C                                                                           RAND 530

      SUBROUTINE RANDU(IX,IY,YFL)                                           RAND 540
      IY=IX*65539                                                           RAND 550
      IF(IY)5,6,6                                                           RAND 560
    5 IY=IY+2147483647+1                                                    RAND 570
    6 YFL=IY                                                                RAND 580
      YFL=YFL*.4656613E-9                                                   RAND 590
      RETURN                                                                RAND 600
      END                                                                   RAND 610
```

5. The Input Data Files Used in the Experiment:

p garray hi45,0a Axisymmetric, 60 psia, z= .05 cm.

```
  450  100    1    2  100   50    1

 0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.06
 1.08   2.28   3.28   4.26   5.10   6.04   6.88   7.76   8.42   9.14
09.50  09.80  10.15  10.60  10.95  11.30  11.60  11.94  12.15  12.42
12.70  12.90  13.10  13.30  13.40  13.55  13.65  13.80  13.87  13.92
14.05  14.15  14.20  14.22  14.28  14.32  14.38  14.40  14.42  14.42

R; T=0.03/0.10 17.28.59
```

p garry@ay hi45,2.5 Axisymmetric, 60 psia, z= .25 cm.

```
  452  100    1    2  100   50    1

 0.00   0.00   0.00   0.00   0.03   0.24   0.62   1.26   1.64   1.90
 2.10   2.32   2.62   2.91   3.40   3.90   4.43   5.03   5.50   6.00
 6.51   6.98   7.35   7.70   8.02   8.44   8.90   9.32   9.03   9.91
10.18  10.40  10.60  10.84  11.04  11.20  11.35  11.50  11.65  11.70
12.00  12.09  12.15  12.21  12.25  12.29  12.32  12.36  12.38  12.39

R; T=0.03/0.10 17.30.09
```

p garray hi45,5a Axisymmetric, 60 psia, z= .5 cm.

```
  542  100    1    2  100   50    1
  5.0
 0.00   0.00   0.03   0.32   0.82   1.33   1.93   2.19   2.30   2.44
 2.53   2.63   2.72   2.79   2.88   3.00   3.15   3.36   3.60   3.80
 4.00   4.28   4.64   4.94   5.20   5.53   5.90   6.18   6.44   6.70
 6.95   7.19   7.40   7.61   7.80   7.97   8.12   8.22   8.48   8.62
 8.75   8.88   8.98   9.10   9.19   9.28   9.37   9.42   9.44   9.46

R; T=0.03/0.10 17.31.06
```

p garray hi45,7.5 Axisymmetric, 60 psia, z= .75 cm

```
 455  120    1    2  100   60    1

0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.18   0.38
0.68   0.98   1.45   1.75   2.28   2.55   2.73   2.84   2.95   3.02
3.08   3.10   3.11   3.12   3.13   3.11   3.09   3.06   3.06   3.12
3.25   3.40   3.59   3.75   3.93   4.10   4.30   4.50   4.66   4.83
5.00   5.13   5.35   5.50   5.65   5.85   6.03   6.17   6.28   6.38
6.50   6.58   6.64   6.72   6.78   6.84   6.88   6.90   6.94   6.96
```

R; T=0.03/0.10 17.32.03

p garray hi45,10. Axisymmetric, 60 psia, z= 1.0 cm.

```
4510  120    1    2  100   60    1
1.0
0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.15   0.38   0.65
1.04   1.30   1.60   2.10   2.37   2.62   2.78   2.91   3.03   3.15
3.22   3.30   3.34   3.36   3.34   3.28   3.20   3.12   3.05   2.98
2.92   2.90   2.90   2.90   2.92   2.97   3.00   3.06   3.12   3.18
3.28   3.35   3.42   3.52   3.63   3.74   3.81   3.90   3.99   4.05
4.20   4.27   4.22   4.27   4.32   4.37   4.40   4.42   4.48   4.50
```

R; T=0.03/0.10 17.33.11

p garray hi45,12. Axisymmetric, 60 psia, z= 1.25 cm.

```
4512  120    1    2  100   60    1
1.25
0.00   0.00   0.00   0.00   0.00   0.04   0.20   0.44   0.70   1.10
1.55   1.88   2.18   2.36   2.52   2.62   2.77   2.87   2.94   3.02
3.00   3.16   3.20   3.24   3.20   3.16   3.11   3.01   2.88   2.70
2.56   2.47   2.38   2.28   2.21   2.14   2.10   2.05   2.05   2.06
2.10   2.16   2.20   2.25   2.31   2.37   2.43   2.50   2.55   2.60
2.66   2.71   2.78   2.85   2.90   2.95   2.99   3.02   3.05   3.07
```

R; T=0.03/0.10 17.34.27

p garray hi45,15. Axisymmetric, 60 psia, z= 1.5 cm.

```
4515  120    1    2  100   60    1
01.5
0.01   0.02   0.03   0.05   0.08   0.12   0.31   0.59   0.89   1.20
1.68   2.09   2.31   2.50   2.61   2.72   2.80   2.86   2.90   2.95
2.98   3.00   2.99   2.98   2.94   2.86   2.79   2.68   2.58   2.44
2.28   2.15   2.00   1.85   1.76   1.70   1.63   1.59   1.55   1.52
1.51   1.49   1.48   1.47   1.45   1.46   1.47   1.48   1.49   1.50
1.51   1.52   1.53   1.55   1.57   1.58   1.59   1.60   1.61   1.61
```

R; T=0.03/0.10 17.35.38

p garray hi45,20. Axisymmetric, 60 psia, z= 2.0 cm.

```
4520  120    1    2  100   60    1
2.0
0.00   0.03   0.10   0.20   0.35   0.58   0.79   0.90   1.07   1.30
1.55   1.75   2.00   2.28   2.45   2.60   2.72   2.85   2.95   3.02
3.09   3.11   3.15   3.17   3.18   3.17   3.16   3.10   3.04   2.97
2.89   2.79   2.68   2.54   2.38   2.14   1.87   1.69   1.50   1.35
1.18   1.03   0.92   0.79   0.65   0.58   0.48   0.39   0.34   0.30
0.25   0.21   0.20   0.20   0.19   0.19   0.18   0.18   0.18   0.17
```

R; T=0.03/0.10 17.36.38

p garray hia,60 Asymmetric, 60 psia, z= .5 cm.

6	6	36	1	1	6	9			
.5									
0.00	0.00	0.00	0.00	0.00	0.10	0.50	1.45	2.03	2.09
2.10	2.30	2.65	3.19	3.70	4.33	4.90	5.39	5.89	6.35
6.88	7.20	7.62	8.04	8.38	8.60	8.75	8.88	8.92	8.97
8.94	8.89	8.78	8.69	8.30	7.96	7.60	7.15	6.63	6.10
5.53	5.08	4.50	3.93	3.38	3.00	2.89	2.75	2.62	2.18
1.78	1.16	0.40	0.06	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.37	0.80	1.49	2.12	2.62
2.89	3.00	3.03	3.13	3.50	3.98	4.52	5.00	5.56	6.08
6.58	6.97	7.44	7.84	8.21	8.56	8.85	9.10	9.30	9.40
9.40	9.32	9.20	9.01	8.76	8.45	8.17	7.86	7.54	7.14
6.64	6.12	5.50	5.02	4.53	4.02	3.61	3.13	2.85	2.63
2.43	2.08	1.88	0.80	0.32	0.05	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.43	0.96	1.52	2.08	2.44
2.60	2.71	2.86	3.02	3.26	3.52	3.85	4.25	4.64	5.10
5.52	5.95	6.35	6.72	7.10	7.38	7.60	7.80	7.95	8.08
8.16	8.20	8.22	8.18	8.07	7.87	7.62	7.32	7.00	6.60
6.22	5.78	5.30	4.72	4.17	3.70	3.34	3.01	2.75	2.52
2.30	2.04	1.64	0.95	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.60	1.35	2.00	2.37	2.60
2.78	2.96	3.11	3.28	3.44	3.70	4.10	4.62	5.22	5.77
6.28	6.70	7.14	7.50	7.86	8.20	8.50	8.72	8.93	9.05
9.10	9.08	9.00	8.89	8.75	8.50	8.24	7.95	7.55	7.12
6.65	6.12	5.60	5.12	4.62	4.10	3.62	3.30	3.10	2.86
2.66	2.34	1.87	1.00	0.22	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.40	1.10	1.73	2.23	2.51
2.60	2.63	2.77	3.16	3.76	4.38	4.95	5.47	6.01	6.57
7.10	7.66	8.03	8.35	8.62	8.89	9.13	9.33	9.53	9.69

9.72	9.70	9.65	9.50	9.22	8.90	8.52	8.15	7.73	7.32
6.88	6.32	5.80	5.13	4.30	3.77	3.30	2.89	2.50	2.10
1.65	1.12	0.28	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.15	0.90	1.77	2.16	2.51	2.71
2.72	2.70	2.88	3.14	3.61	4.14	4.70	5.24	4.77	6.28
6.75	7.20	7.60	8.00	8.33	8.63	8.90	9.13	9.34	9.51
9.64	9.62	9.57	9.45	9.30	9.09	8.81	8.48	8.07	7.63
7.11	6.52	5.98	5.32	4.70	4.16	3.60	3.05	2.76	2.55
2.21	1.47	0.67	0.05	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.32	1.00	1.83	2.31	2.59	2.58
2.55	2.59	2.78	3.05	3.45	3.97	4.52	5.05	5.56	6.11
6.62	7.10	7.55	7.95	8.32	8.62	9.00	9.28	9.50	9.62
9.69	9.69	9.65	9.53	9.34	9.10	8.90	8.70	8.38	8.00
7.58	7.17	6.59	5.80	5.17	4.54	3.90	3.37	2.94	2.53
2.12	1.50	0.84	0.10	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.50	1.20	1.84	2.28	2.40	2.30
2.30	2.42	2.67	2.97	3.38	3.82	4.32	4.78	5.30	5.82
6.29	6.78	7.00	7.33	7.70	8.02	8.35	8.62	8.90	9.09
9.21	9.19	9.10	9.00	8.86	8.65	8.42	8.10	7.75	7.38
6.92	6.45	5.96	5.42	4.78	4.17	3.40	2.87	2.53	2.18
1.69	1.06	0.38	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.03	0.36	0.80	1.25	1.61	1.92	2.22	2.38
2.38	2.30	2.38	2.70	3.02	3.50	3.93	4.42	5.01	5.56
5.95	6.35	6.72	7.03	7.45	7.77	8.08	8.32	8.50	8.62
8.70	8.68	8.60	8.50	8.35	8.20	7.98	7.70	7.38	7.00
6.64	6.20	5.70	5.12	4.52	3.89	3.25	2.62	2.25	2.00
1.70	1.17	0.47	0.09	0.00	0.00	0.00	0.00	0.00	0.00

R; T=0.09/0.60 17.44.25

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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)
Naval Postgraduate School
Monterey, California 93940

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b. GROUP

3. REPORT TITLE
The Application of Holographic Interferometry to the Determination of Asymmetric Three-Dimensional Density Fields in Free Jet Flow

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Ph. D. Thesis (June 1970)

5. AUTHOR(S) (First name, middle initial, last name)
Robert Dale Matulka

6. REPORT DATE
June 1970

7a. TOTAL NO. OF PAGES
156

7b. NO. OF REFS
37

8a. CONTRACT OR GRANT NO.
NAVAIR A33-330-551-69

b. PROJECT NO.

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

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12. SPONSORING MILITARY ACTIVITY
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13. ABSTRACT

The successful application of holographic interferometry, and an associated mathematical reduction process, to the determination of an asymmetric three-dimensional density field of an aerodynamic phenomenon is reported.

An integral inversion method from the field of plasma physics has been computerized, extensively evaluated and applied to the determination of functions, both axisymmetric and asymmetric, which simulate aerodynamic density fields.

The application of holographic interferometry has been extended to provide multiple holograms about a test region, with sufficient coverage to provide interferometric data for the successful solution of the density field.

The analytical and experimental methods developed were applied to an experimental axisymmetric test field, the supersonic flow from a free jet, and shown to be comparable to a previous solution obtained by the Abel inversion method. Further, the free jet was tilted to provide a test field which was asymmetric in the plane of solution. Comparison of the resulting asymmetric solution was shown to be self-consistent with the previously obtained axisymmetric solution.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
1. Holography						
2. Interferometry						
3. Three-Dimensional						
4. Asymmetric						
5. Three-Dimensional Density Measurement						
6. Asymmetric Field Solution						

Thesis 122517
M3835 Matulka
c.1 The application of holographic interferometry to the determination of asymmetric three-dimensional density fields in free jet flow.